^{3}He structure and Generalized Parton Distributions

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Outline

- Hard exclusive processes
  and Generalized Parton Distributions (GPDs)

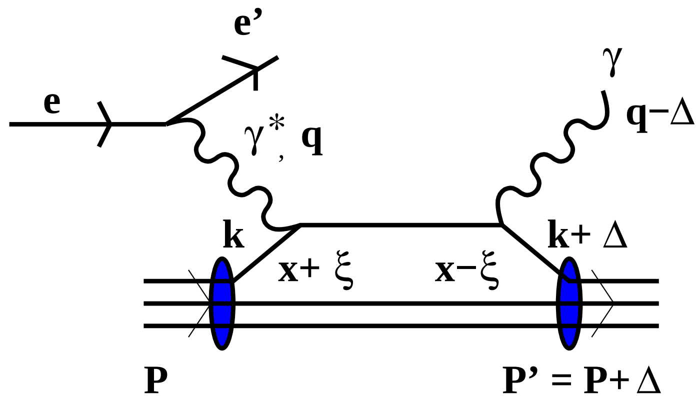
- GPDs of nuclei:
  - GPDs in Impulse Approximation (IA);
  - relevance of light nuclei.

- Calculation of GPDs of ^{3}He
  (S.S., PRC 70 (2004) 015205; NPA 755c (2005) 523)

- Conclusions

GPDs: Definition (X. Ji PRL 78 (97) 610)

For a $J = \frac{1}{2}$ target,
in a hard-exclusive process,
$(Q^2, \nu \to \infty)$
such as DVCS:



the GPDs $H_q(x, \xi, \Delta^2)$ and $E_q(x, \xi, \Delta^2)$ are introduced:

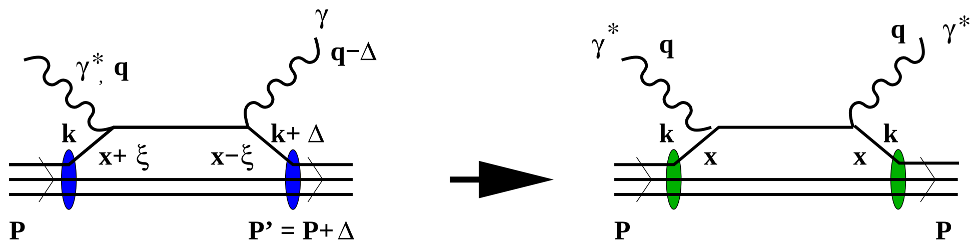
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \quad \gamma^\mu \quad \psi_q(\lambda n/2) | P \rangle = H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P)$$

$$+ \quad E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \ldots$$

- $\Delta = P' - P$, $q^\mu = (q_0, \vec{q})$, and $\bar{P} = (P + P')^\mu / 2$
- $x = k^+ / P^+$; ξ = "skewness" = $-\Delta^+ / (2\bar{P}^+)$
- $x \leq -\xi \longrightarrow$ GPDs describe $antiquarks$;
  $-\xi \leq x \leq \xi \longrightarrow$ GPDs describe $q\bar{q}$ $pairs$; $x \geq \xi \longrightarrow$ GPDs describe $quarks$

GPDs: limits

- when $P' = P$, i.e., $\Delta^2 = \xi = 0$, one recovers the usual PDFs:



$$H_q(x,\xi,\Delta^2) \Longrightarrow H_q(x,0,0) = q(x)$$

- the x–integration yields the q-contribution to the Form Factors

$$\int dx \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma^\mu \psi_q(\lambda n/2) | P \rangle =$$

$$\int dx\, H_q(x,\xi,\Delta^2) \bar{U}(P') \gamma^\mu U(P) + \int dx\, E_q(x,\xi,\Delta^2) \bar{U}(P') \frac{\sigma^{\mu\nu}\Delta_\nu}{2M} U(P) + \ldots$$

$$\Longrightarrow \int dx\, H_q(x,\xi,\Delta^2) = F_1^q(\Delta^2) \qquad \int dx\, E_q(x,\xi,\Delta^2) = F_2^q(\Delta^2)$$

GPDs: polarized case (X. Ji PRL 78 (97) 610)

For a $J = \frac{1}{2}$ target, in a hard-exclusive process, ($Q^2, \nu \to \infty$) the GPDs $\tilde{H}_q(x,\xi,\Delta^2)$ and $\tilde{E}_q(x,\xi,\Delta^2)$ are introduced:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\bar{\psi}_q(-\lambda n/2) \quad \gamma^\mu \gamma_5 \quad \psi_q(\lambda n/2)|P\rangle = \tilde{H}_q(x,\xi,\Delta^2)\bar{U}(P')\gamma^\mu\gamma_5 U(P)$$

$$+ \quad \tilde{E}_q(x,\xi,\Delta^2)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\gamma_5 U(P) + ...$$

- In the forward limit, $\Delta^2 = \xi = 0$, one recovers the helicity-dependent GPDs:

$$\tilde{H}_q(x,\xi,\Delta^2) \Longrightarrow \tilde{H}_q(x,0,0) = \Delta q(x)$$

- the x–integration yields the q-contribution to the Form Factors

$$\int dx\, \tilde{H}_q(x,\xi,\Delta^2) = G_A^q(\Delta^2)$$

GPDs: A unique tool...

- to unravel the spin structure of hadrons at parton level
  (access to orbital angular momentum, solution (?) of the "Spin Crisis");
- to explore the 3-dimensional structure of hadrons at parton level;
- to access crucial features of small-x Physics
  (vector meson production and gluon distributions).

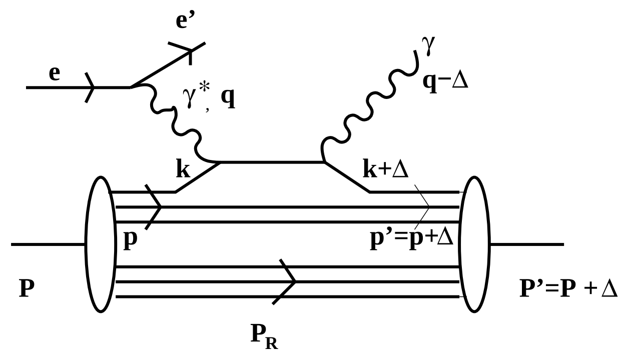
... but also an experimental challenge:

- Hard exclusive processes $\longrightarrow$ small X-sections;
- GPDs enter the X-sections through convolutions in the x variable:

$$T_{DVCS} \propto \int_{1}^{1} dx \, \frac{H(x,\xi,\Delta^2)}{x-\xi+i\epsilon} + \ldots$$

the extraction of the information on GPDs from data is difficult;
The "slice" $x=\xi$ gives the main contribution.

Nuclei: why?



ONE of the reasons is understood by studying coherent DVCS in I.A.:

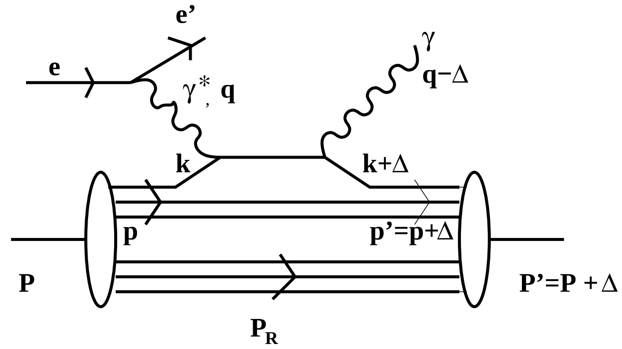
In a symmetric frame ($\bar{p} = (p + p')/2$) :

$$k^+ = (x + \xi)\bar{P}^+ = (x' + \xi')\bar{p}^+ \ ,$$
$$(k + \Delta)^+ = (x - \xi)\bar{P}^+ = (x' - \xi')\bar{p}^+ \ ,$$

if $\Delta^2 \ll M^2, \xi^2 \ll 1$, one has

$$H_q(x, \xi, \Delta^2) \simeq \int d\vec{k}\, \frac{k^+}{k_0}\, \delta\left(x + \xi - \frac{k^+}{\bar{P}^+}\right) \sum_r \langle P' | b_r^{q,+}(\vec{k} + \vec{\Delta}) b_r^q(\vec{k}) | P \rangle \ .$$

Nuclei: why?



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$$

By properly inserting complete sets of states for the interacting nucleon and the recoiling system (S.S., V. Vento PRD 69, (2004) 094004):

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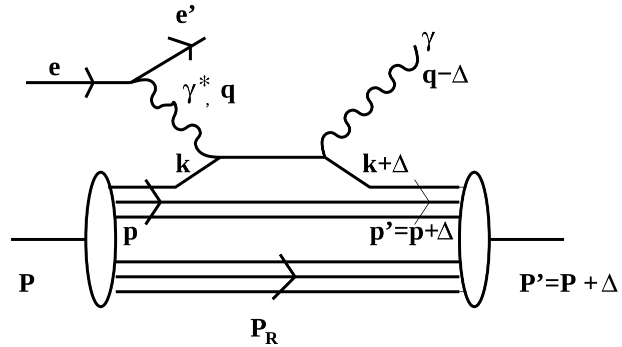

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\end{aligned}
$$

if $\Delta^2 \ll M^2, \xi^2 \ll 1$, one has

$$
H_q(x,\xi,\Delta^2) = \langle P'| \sum_{\vec{P}'_R, S'_R, \vec{p}', s'} \{|P'_R S'_R\rangle |p' s'\rangle\}\{\langle P'_R S'_R| \langle p' s'|\}
$$

$$
\int d\vec{k}\, \frac{k^+}{k_0}\, \delta\left(x + \xi - \frac{k^+}{\bar{P}^+}\right) \sum_r b^+_{q,r}(\vec{k}+\vec{\Delta}) b_{q,r}(\vec{k})
$$

$$
\sum_{\vec{P}_R, S_R, \vec{p}, s} \{|P_R S_R\rangle |ps\rangle\}\{\langle P_R S_R| \langle ps|\} \quad |P\rangle \ ,
$$

Why nuclei? - 2

and, since $\{\langle P_R S_R | \langle ps | \} | PS \rangle = \langle P_R S_R, ps | PS \rangle (2\pi)^3 \delta^3 (\vec{P} - \vec{P}_R - \vec{p}) \delta_{S, S_R\, s}$,
a convolution formula can be obtained (S.S. PRC 70, 015205 (2004)):

$$H_q^A(x,\xi,\Delta^2) \simeq \sum_N \int_x^1 \frac{dz}{z} h_N^A(z,\xi,\Delta^2) H_q^N\left(\frac{x}{z},\frac{\xi}{z},\Delta^2\right)$$

in terms of $H_q^N(x',\xi',\Delta^2)$, the GPD of the nucleon N, and of the light-cone off-diagonal momentum distribution:

$$h_N^A(z,\xi,\Delta^2) = \int dE d\vec{p}\, P_N^A(\vec{p},\vec{p}+\vec{\Delta},E)\delta\left(z+\xi-\frac{p^+}{\bar{P}^+}\right)$$

where $P_N^A(\vec{p},\vec{p}+\vec{\Delta},E)$, is the one-body off-diagonal spectral function for the nucleon N in the nucleus,

$$P_N^3(\vec{p},\vec{p}+\vec{\Delta},E) = \frac{1}{(2\pi)^3}\frac{1}{2}\sum_M \sum_{R,s} \langle \vec{P}' M | (\vec{P}-\vec{p}) S_R, (\vec{p}+\vec{\Delta}) s \rangle$$
$$\times \langle (\vec{P}-\vec{p}) S_R, \vec{p} s | \vec{P} M \rangle\, \delta(E - E_{min} - E_R^*)\ .$$

Why nuclei? - 3

The obtained expressions have the correct limits:

- the x-integral gives the f.f. $F_q^A(\Delta^2)$ in I.A.:

$$\int dx H_q^A(x,\xi,\Delta^2) = F_q^N(\Delta^2) \int dE d\vec{p} P_N^A(\vec{p},\vec{p}+\vec{\Delta},E) = F_q^A(\Delta^2)$$

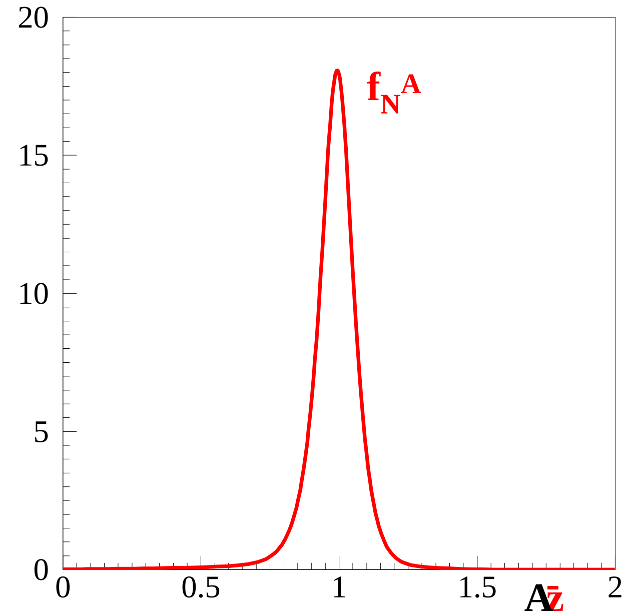
- forward limit (standard DIS):

$q^A(x) \simeq \sum_N \int_x^1 \frac{d\tilde{z}}{\tilde{z}} f_N^A(\tilde{z}) q^N\left(\frac{x}{\tilde{z}}\right)$

with the light-cone momentum distribution:

$f_N^A(\tilde{z}) = \int dE d\vec{p} P_N^A(\vec{p},E)\delta\left(\tilde{z} - \frac{p^+}{P^+}\right)$,

which is strongly peaked around $A\tilde{z} = 1$:



Why nuclei? - 4

In DIS, the contribution of nucleons with large longitudinal momentum $\tilde{z}$ can be observed only at high x, where the cross sections are vanishingly small, since q^N is also vanishing; the same does not happen for GPDs, since one has:

$$H_q^A(x,\xi,\Delta^2) \simeq \sum_N \int_x^1 \frac{dz}{z} h_N^A(z,\xi,\Delta^2) H_q^N\left(\frac{x}{z},\frac{\xi}{z},\Delta^2\right) \qquad \text{with:}$$

$$h_N^A(z,\xi,\Delta^2) = \int dE d\vec{p}\, P_N^A(\vec{p},\vec{p}+\vec{\Delta},E)\delta\left(z+\xi-\frac{p^+}{\bar{P}^+}\right) \ ,$$

so that, by tuning the independent variable ξ,
one can select regions where nucleons with large
$\tilde{z}$ dominate, even if x is low (being $x \leq z = \tilde{z} - \xi$)
and H_q^N is not vanishing.

(Firstly observed in:
Berger, Cano, Diehl and Pire,
PRL 87 (2001) 142302)

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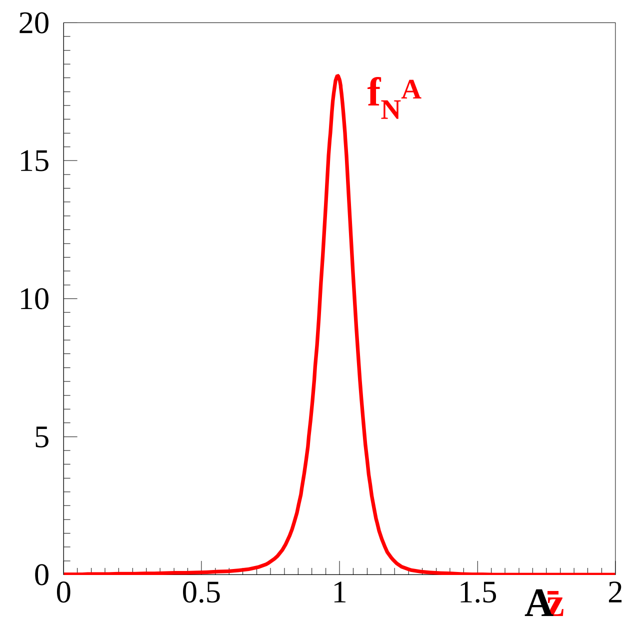

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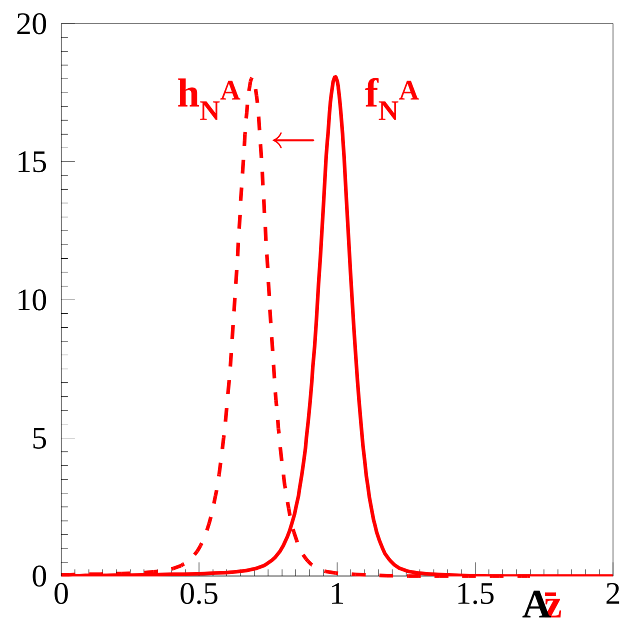


Why nuclei? - 5

Several relevant issues can be therefore investigated...:

- the nuclear short range structure, at quark level, can be accessed through non-vanishing observables;
- the reaction mechanism, i.e. the validity of I.A. and the relevance of effects beyond it (non nucleonic degrees of freedom, off-shell effects...);
- other features not discussed here: large distance nuclear structure (Freund and Strikman, PRC 69 (2004) 015203); transverse location of quarks in nuclei (Ralston & Pire PRD 66 (2002) 111501; Polyakov PLB 555 (2003) 57); Color Transparency phenomena, (S.Liuti and S.K.Taneja, hep-ph/0504027); ^{4}He and heavier spinless nuclear targets (Guzey & Strikman, PRC 68 (2003) 015204);

... with some effort:

measurements are difficult: need for a recoil detector to be sure that the nucleus did not break (despite of this, some data are already available!);

GPDs for light nuclei - the deuteron:

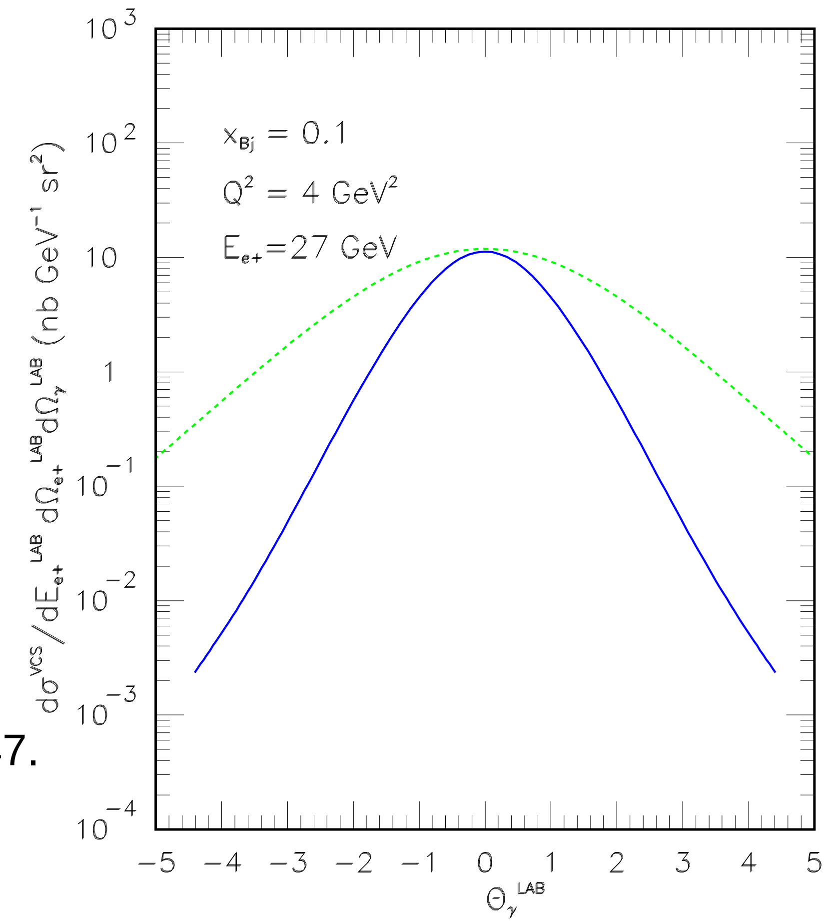
In this framework, as always, light nuclei play a special rôle: conventional nuclear effects can be safely estimated for them. Any deviation from the predicted behavior can be interpreted as an exotic one.

For the deuteron a complete analysis is being done:

- for spin 1 nuclei, GPDs have been defined;
  Berger, Cano, Diehl, Pire, PRL 87 (2001) 142302
- for the deuteron, GPDs have been calculated in I.A., using a light-front approach;
  Cano & Pire, NPA 711 (2002) 133
- for the coherent (no break-up) channel, the relevant cross sections and asymmetries have been estimated, for photon and meson deep electroproduction, in the kinematics of JLAB and HERMES;

The deuteron - 2

The signal of coherent scattering on the deuteron is found to be comparable to that found for the proton target, at least at low momentum transfer → experiments are feasible and the short distance structure of the deuteron can be studied.



Cano & Pire, EPJA 19 (2004) 423;
Kirchner & Müller, EPJC 32 (2003) 347.

GPDs for ^{3}He

^{3}He is a theoretically well known nucleus and it has been used extensively as an effective neutron target, especially to unveil the spin structure of the free neutron. The helicity independent GPD H_q^3 has been calculated in I.A., in the $\Delta^2 \ll M^2$, $\xi^2 \ll 1$ region, relevant to the study of coherent DVCS.

In the convolution formula, valid for any spin 1/2 nucleus:

$$H_q^3(x,\xi,\Delta^2) \simeq \sum_N \int_x^1 \frac{dz}{z} h_N^3(z,\xi,\Delta^2) H_q^N\left(\frac{x}{z},\frac{\xi}{z},\Delta^2\right)$$

the light-cone off-diagonal momentum distribution:

$$h_N^3(z,\xi,\Delta^2) = \int dE d\vec{p}\, P_N^3(\vec{p},\vec{p}+\vec{\Delta},E)\delta\left(z+\xi-\frac{p^+}{\bar{P}^+}\right)$$

has been calculated evaluating the one-body off-diagonal spectral function $P_N^3(\vec{p},\vec{p}+\vec{\Delta},E)$, by means of a realistic treatment based on wave functions obtained through the AV18 interaction (w.f. from the Pisa group, A. Kievsky *et al* NPA 577, 511 (1994), overlaps from the Pisa-Roma collaboration, A. Kievsky *et. al*, PRC 56, 64 (1997)). For H_q^N, a model by Radyushkin (PRD 61, 074027 (2000)) has been used.

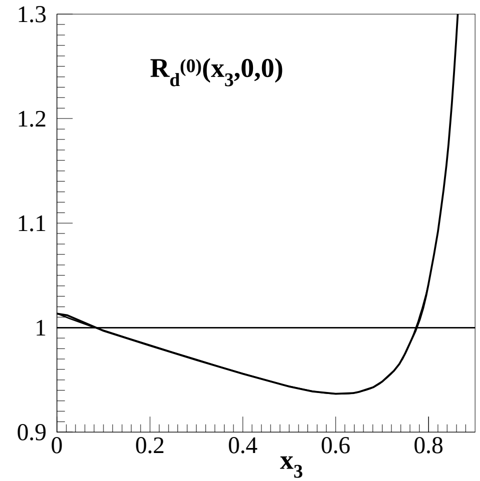
The calculation has the correct limits:

1 - Forward limit: the ratio:

$$R_q(x,0,0) = \frac{H_q^3(x,0,0)}{2H_q^p(x,0,0)+H_q^n(x,0,0)}$$

$$= \frac{q^3(x)}{2q^p(x)+q^n(x)}$$

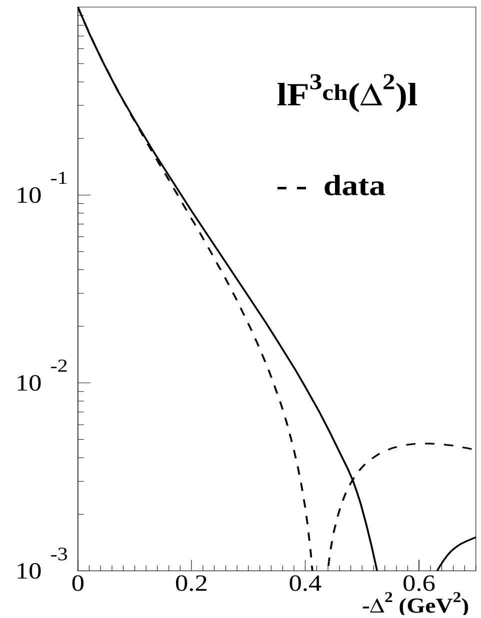
shows an EMC-like behavior;



2 - Charge F.F.:

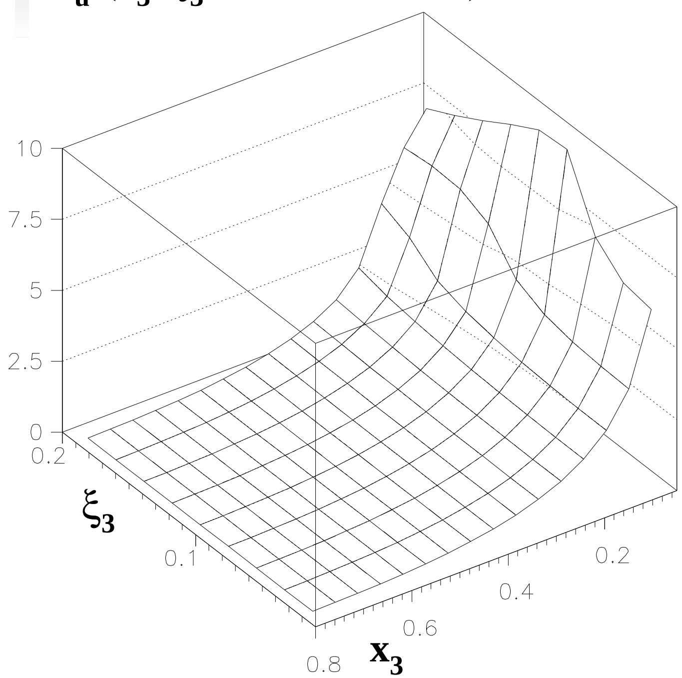
$$\int dx H_q^3(x,\xi,\Delta^2) = F_q^3(\Delta^2)$$

in good agreement with data in the region relevant to the coherent process, $\Delta^2 \ll 0.25$ GeV^2.



Typical trend ($x_3 = 3\,x, \xi_3 = 3\,\xi$)

$H_u^3(x_3, \xi_3, \Delta^2\text{=-0.15 GeV}^2)$



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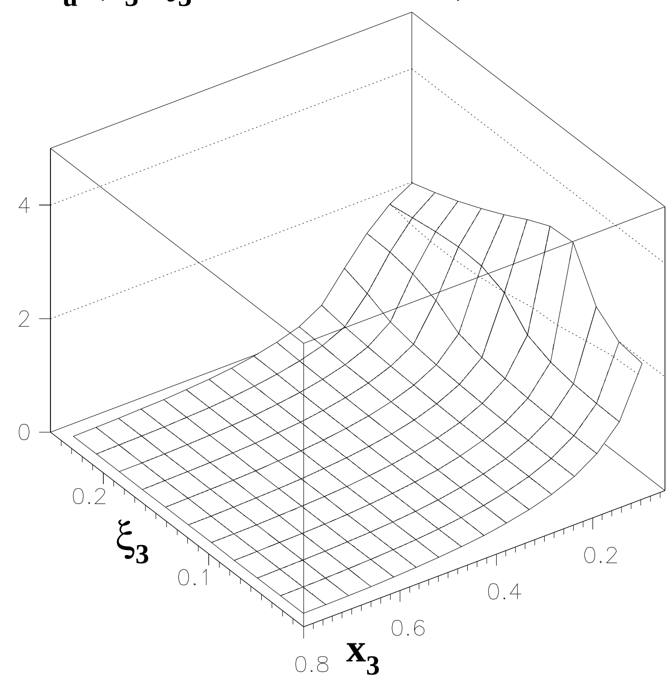
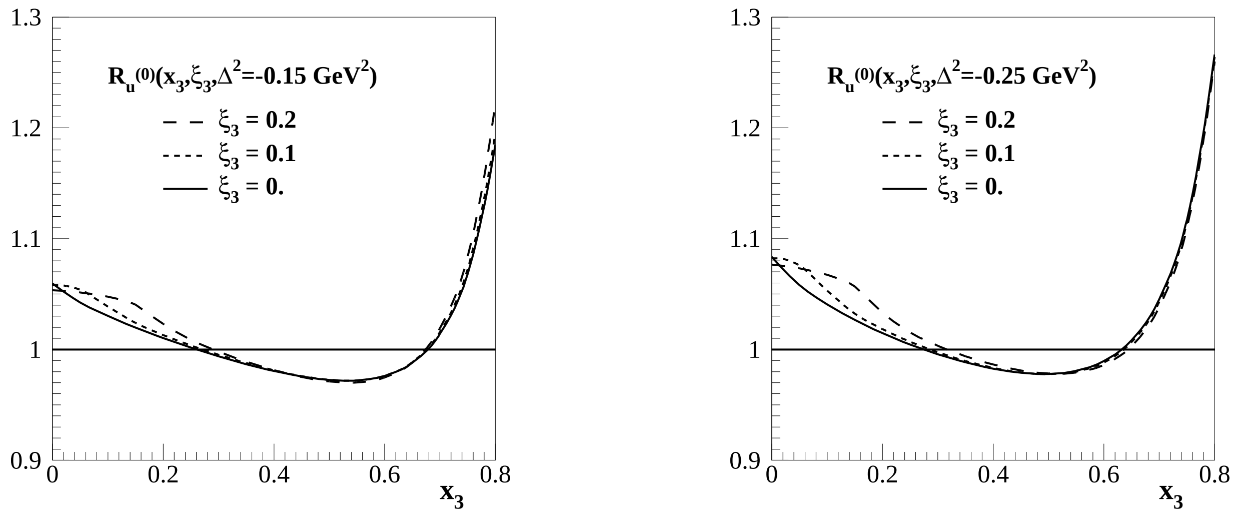


Nuclear effects - 1

- Nuclear effects grow with ξ at fixed Δ^2, and with Δ^2 at fixed ξ:



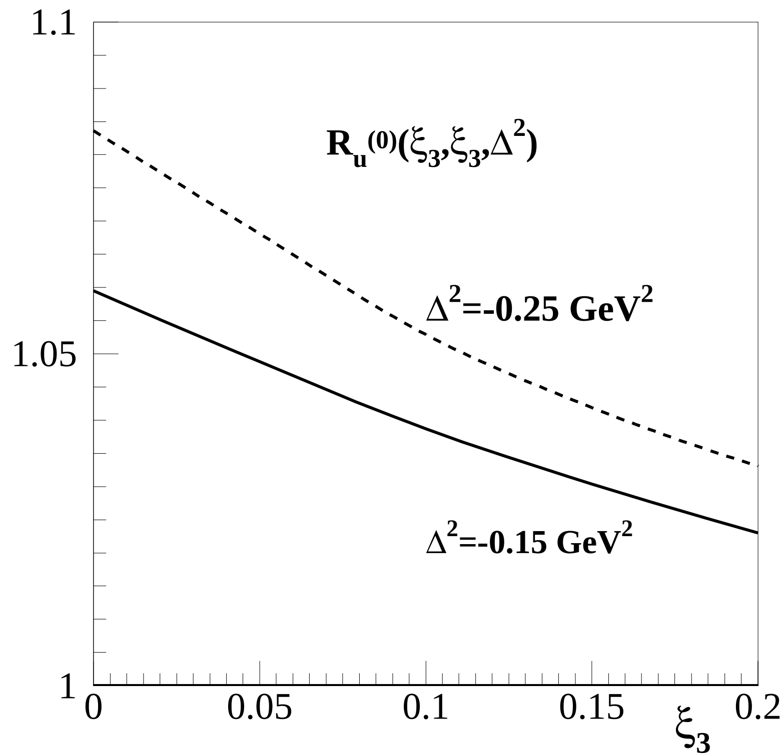
$$R_q^{(0)}(x,\xi,\Delta^2) = \frac{H_q^3(x,\xi,\Delta^2)}{2H_q^{3,p}(x,\xi,\Delta^2) + H_q^{3,n}(x,\xi,\Delta^2)}$$

$$H_q^{3,N}(x,\xi,\Delta^2) = \tilde{H}_q^N(x,\xi)F_q^3(\Delta^2)$$

$R_q^{(0)}(x,\xi,\Delta^2)$ would be one if there were no nuclear effects;
as it is found also for the deuteron, there is no factorization into terms dependent separately on Δ^2 and x, ξ (the factorization hypotheses has been used to estimate nuclear GPDs).

Nuclear effects - 2

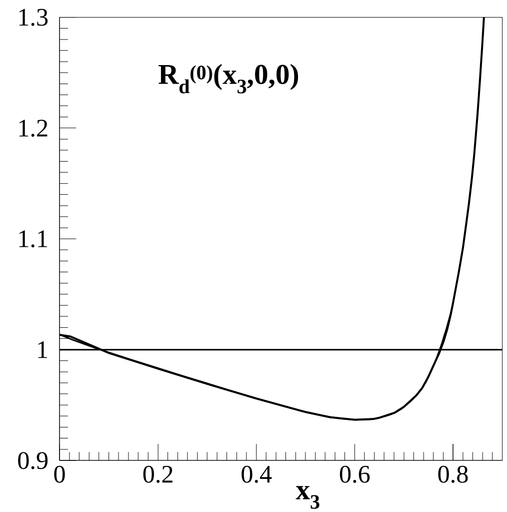
- Nuclear effects are large even in the important region $x = \xi$:



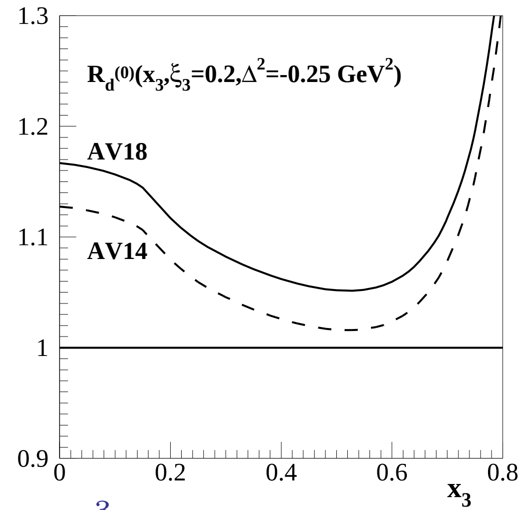
Nuclear effects - 3

Nuclear effects are bigger than in the forward case: dependence on the potential

- Forward case: Calculations using the AV14 or AV18 interactions are indistinguishable



- Non-forward case: Calculations using the AV14 and AV18 interactions do differ:



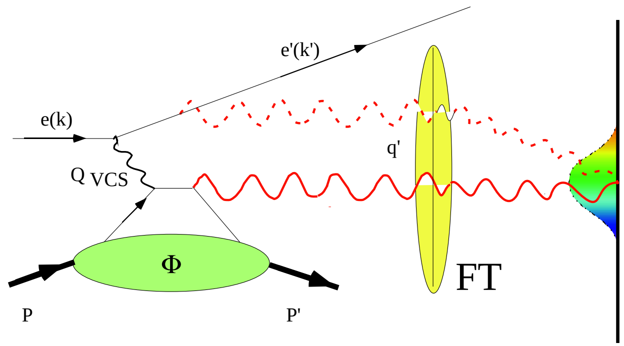
Conclusions

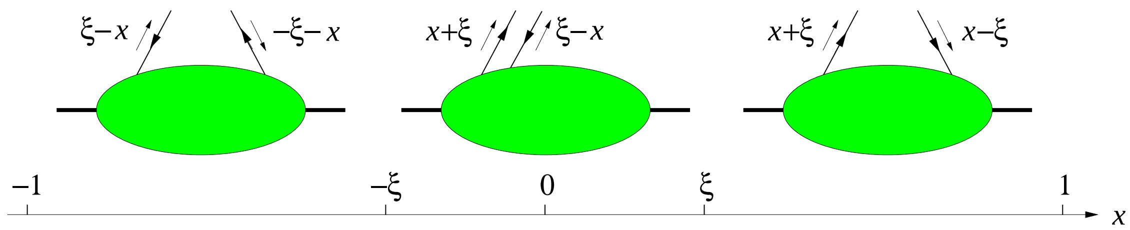
- What has been done:
  - An instant form, I.A. calculation of $H_q(x, \xi, \Delta^2)$, using the AV18 wave functions;
  - Estimates of nuclear effects: larger than in the forward situation, increasing with ξ and Δ^2, larger for the d flavor than for the u one, important also at $x = \xi$, dependent on the NN interaction.

- What has to be done:
  - to implement a relativistic treatment,
  - to estimate X-sections and helicity dependent GPDs $\longrightarrow$ $\longrightarrow$ angular momentum structure of the free neutron.

- What is expected:
  - To get a short distance understanding of quark confinement in light nuclei;
  - To obtain a picture of light nuclei at quark level;
  - To understand to what extent light nuclei are loosely bound systems of protons and neutrons.

some figures:





some figures - 2: