The Electrodisintegration of $^3\vec{\mathrm{He}}$ and the role of FSI at Jlab energies

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Outline

- Motivations
- Em Responses of $^3\vec{\mathrm{He}}$ within the Plane Wave Impulse Approximation
- Final state interaction: three-nucleon states in the continuum
- Conclusions &Perspectives

Preliminary results in EPJA **19** (2004) 87

Motivations

- $^3\vec{\mathrm{He}}$ as an effective neutron target $\rightarrow$ elastic and inelastic em responses of the neutron
- An attempt to include in a non perturbative way relativistic features (requested both by kinematics and by Poincaré covariance..)
- Test for the present-day, accurate wave functions of the three-nucleon system : bound and continuum w.f.'s

Caveats: isobar configurations, MEC ....

As a first step, the analysis of inclusive em responses of polarized ^{3}He was carried out within the *plane wave impulse approximation* (PWIA)$\equiv$ $|\mathrm{plane\ w.}\rangle_1\ |\mathrm{interact.\ w.f.}\rangle_{23}$

Em Responses of $^3\vec{\mathrm{He}}$ within the PWIA

Approximations

- Relativistic kinematics but no Wigner Functions for boosting initial and final wave functions of the three-nucleon system (as a straightforward improvement we will extend our calculation to the Front Form of the Relativistic Hamiltonian Dynamics, as we did for the elastic form factor of the Deuteron, PRC 62 (2000) 0640004)

$$\langle j', j'_z; \epsilon'_{int}, \alpha'_i, \mathbf{q}| J^\mu_{IA}(0)|\frac{1}{2}, j_z; \epsilon_{int}, \alpha_i, \mathbf{0}\rangle =$$

$$= 3\int d\mathbf{k}_1 d\mathbf{k}_2 \langle j', j'_z; \epsilon'_{int}, \alpha'_i|\mathbf{k}'_1, \mathbf{k}'_2\rangle \quad \leftarrow (excited\ state)$$

$$\times \langle \mathbf{q}+\mathbf{k}_1|J^\mu_{1,free}(0)|\mathbf{k}_1\rangle \quad \leftarrow (1b - current)$$

$$\times \langle \mathbf{k}_1, \mathbf{k}_2|\frac{1}{2}, j_z; \epsilon_{int}, \alpha\rangle \quad \leftarrow (bound\ state)$$

- Relativistic electron-nucleon cross section $\rightarrow$ CC1 by T. De Forest, NPA **392**, 232 (1983)

- Bound state of ^{3}He $\rightarrow$ solution of a Schrödinger Equation (with, e.g., Argonne V18 $\rightarrow$ R. B. Wiringa, R. A. Smith and T. A. Ainsworth, PRC 29, 1207 (1984))

- Excited states of the three-nucleon system is approximated by (*this is the core of the PWIA !*)

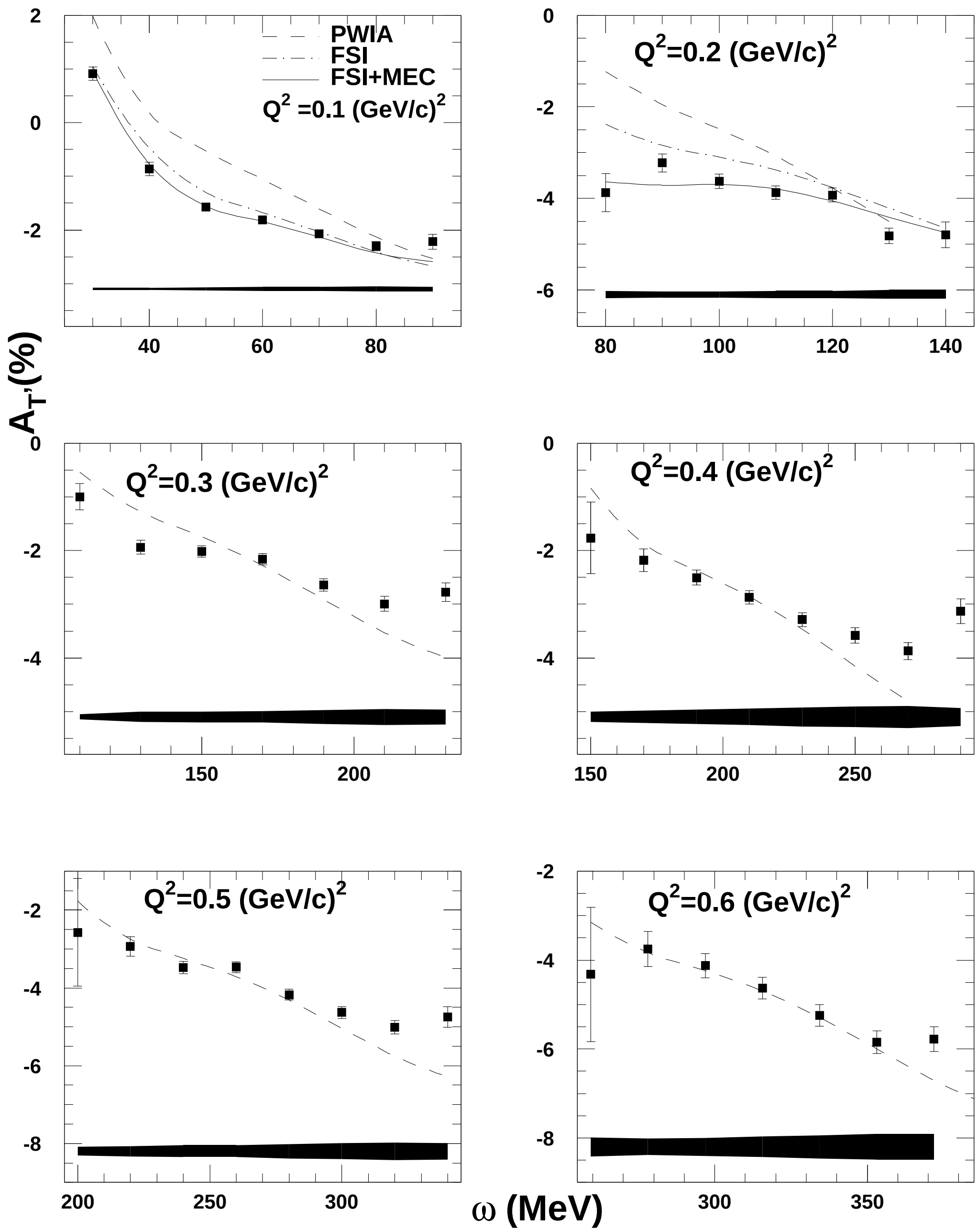
$$|j', j'_z; \epsilon'_{int}, \alpha'; \mathbf{q}\rangle \to \frac{1}{\sqrt{3}}|\mathbf{p}_f, \sigma_f \tau_f\rangle |j_{23}\mu_{23}, \pi_{23}, T_{23}\tau_{23}, \epsilon_{23}; \mathbf{P}_{23}\rangle$$

where

- $\mathbf{p}_f + \mathbf{P}_{23} = \mathbf{q}$
- $|\mathbf{p}_f, \sigma_f \tau_f\rangle \equiv$ Plane wave describing the struck nucleon
- $|j_{23}\mu_{23}, \pi_{23}, T_{23}\tau_{23}, \epsilon_{23}; \mathbf{P}_{23}\rangle \equiv$ *fully-interacting* two-body wave function (describing the spectator pair)
- terms that properly antisymmetrize the three-nucleon wave function are dropped out, and only the direct interaction between the virtual photon and the struck nucleon is taken into account

$$\vec{e} + {}^3\vec{He} \rightarrow e' + X$$

$$A = \frac{\sigma(\uparrow\rightarrow) - \sigma(\uparrow\leftarrow)}{\sigma(\uparrow\rightarrow) + \sigma(\uparrow\leftarrow)} \qquad \theta^* = 0^0 \;\rightarrow\; A_{T'}$$



■ ≡ TJLAB data

Solid lines: Bochüm calculations with fully-interacting three-nucleon w.f. + two-body currents, non-relativistic

Dashed lines: Rome-Pisa PWIA + relativistic σ_{eN} and kinematics

(After Xu et al , Phys.Rev.Lett. 85 (2000) 2900)

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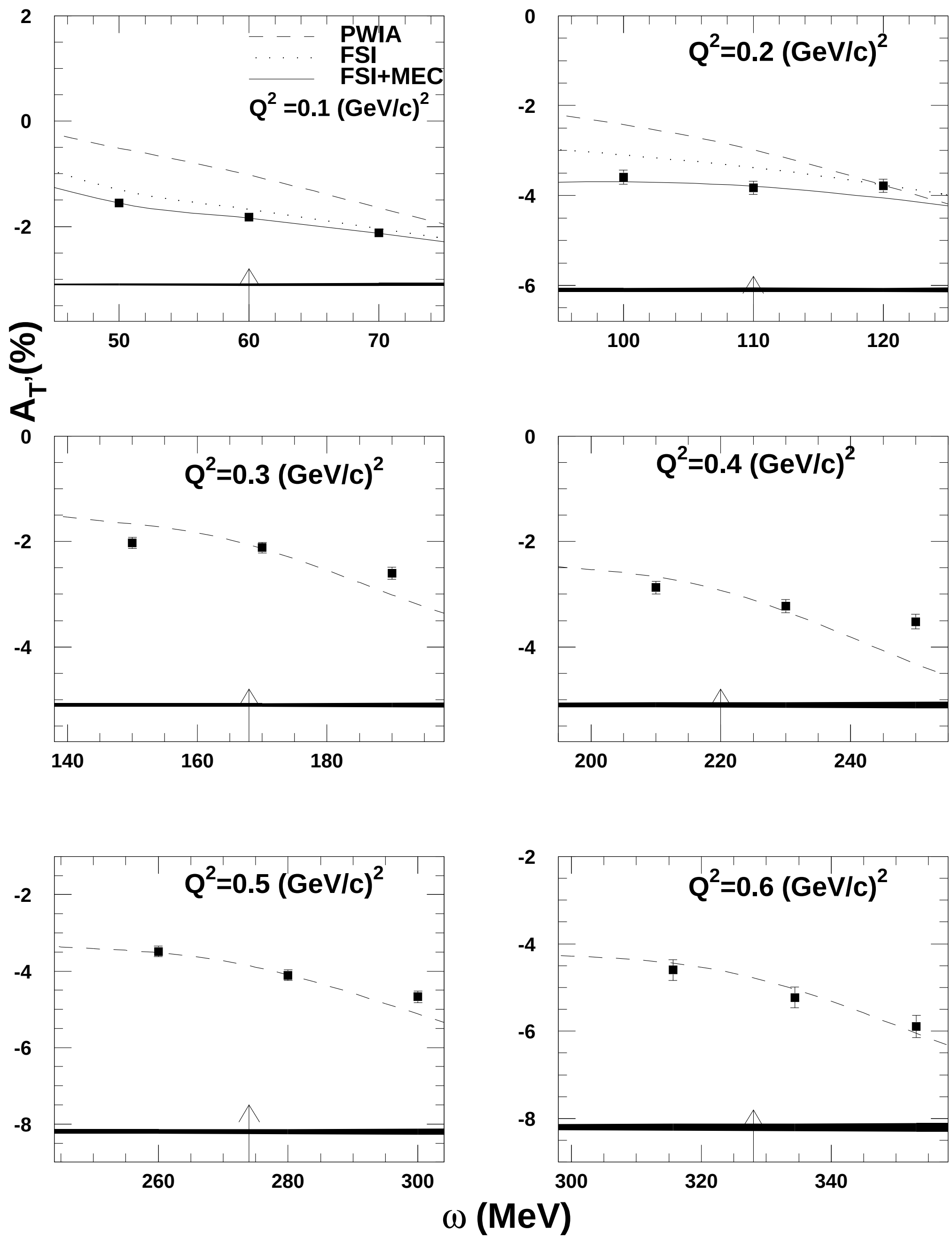

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(After Xu et al , PRC **67** (2003) 012201)

Fully-interacting excited states for a three-nucleon system

The fully-interacting state for a three-nucleon system in the continuum can be decomposed (Kievsky, Rosati Viviani, NPA 577 (1994) 511) as follows

$$\Phi^{LXjj_zTT_z} = \Psi_A^{LXjj_zTT_z} + \Psi_c^{LXjj_zTT_z} =$$

$$= \sum_{i=1}^{3} \left[\psi_A^{LXjj_zTT_z}(i) + \psi_c^{LXjj_zTT_z}(i) \right]$$

where $\psi_A^{LXjj_zTT_z}(i)$ and $\psi_c^{LXjj_zTT_z}(i)$ are three Faddeev-like amplitudes, corresponding to the three permutations of the intrinsic coordinates ($\equiv \{\mathbf{r_1},\ \mathbf{r_2},\ \mathbf{r_3}\}$).

- $\Psi_c^{LXjj_zTT_z}$ describes the system when the three nucleons are close each other. For large interparticle distances and energies below the Deuteron breakup threshold $\Psi_c^{LXjj_zTT_z}$ goes to zero, while for higher energies, $\Psi_c^{LXjj_zTT_z}$ must reproduce an outgoing three-particle state .
- $\Psi_A^{LXjj_zTT_z}$ is the solution of the Schödinger eqn. in the asymptotic region.

★ $\Psi_A^{LXjj_zTT_z}$ can be recast in a different way, in order to emphasize its physical content.

1. the first one produces the PWIA, i.e. contains an interacting pair times a free particle (for the sake of concreteness the particle 1),
2. the second term describes the rescattering between the interacting pair and the particle 1
3. the third term ($\psi_A^{LXjj_zTT_z}(2) + \psi_A^{LXjj_zTT_z}(3)$) takes care of the correct antisymmetrization of $\Psi_A^{LXjj_zTT_z}$ itself.

As an example $\psi_A^{LXjj_zTT_z}(1)$ can be written

$$\psi_A^{LXjj_zTT_z}(1) = \overbrace{\Omega^R_{LXJ}(\mathbf{x}_1, \mathbf{y}_1)}^{PWIA} +$$
$$\underbrace{\sum_{L'X'} {}^J\mathcal{L}^{XX'}_{LL'} \left[\imath\, \Omega^R_{L'X'J}(\mathbf{x}_1, \mathbf{y}_1) + \Omega^I_{L'X'J}(\mathbf{x}_1, \mathbf{y}_1)\right]}_{rescattering}$$

- $\mathbf{x}_1 = \mathbf{r}_2 - \mathbf{r}_3 \qquad \mathbf{y}_1 = [\mathbf{r}_2 + \mathbf{r}_3 - 2\mathbf{r}_1]/\sqrt{3}$, L is the angular orbital momentum of the third particle and X an intermediate momentum coupling between the spin of the third particle and the total momentum of the pair.
- $\Omega^{R(I)}_{LSJ}$ is the regular ("irregular") solution describing the free scattering of a nucleon by an interacting pair (in our first application of the method: a deuteron);

- The matrix

$$\mathcal{L} = \frac{S-1}{2\imath} = -\pi T$$

is related to the Scattering matrix and represents a key ingredient of the variational approach for obtaining the whole wave function.

★ $\Psi_c^{LXjj_zTT_z}$ is explicitly expanded on the complete set of Hyperspherical Harmonics Polynomials, with the inclusion of proper pair-correlation function

★★ The wave function, (i.e. the matrix elements of the S-matrix and the correlation functions), comes out from a variational approach applied to a functional constructed from the expectation value of the S-matrix (complex Köhn variational principle, Kievsky, Rosati Viviani FBS **30**(2001) 39).

As a first step, the excited states with a $|p\ d\rangle$ incoming state has been included in the calculation of the em responses.

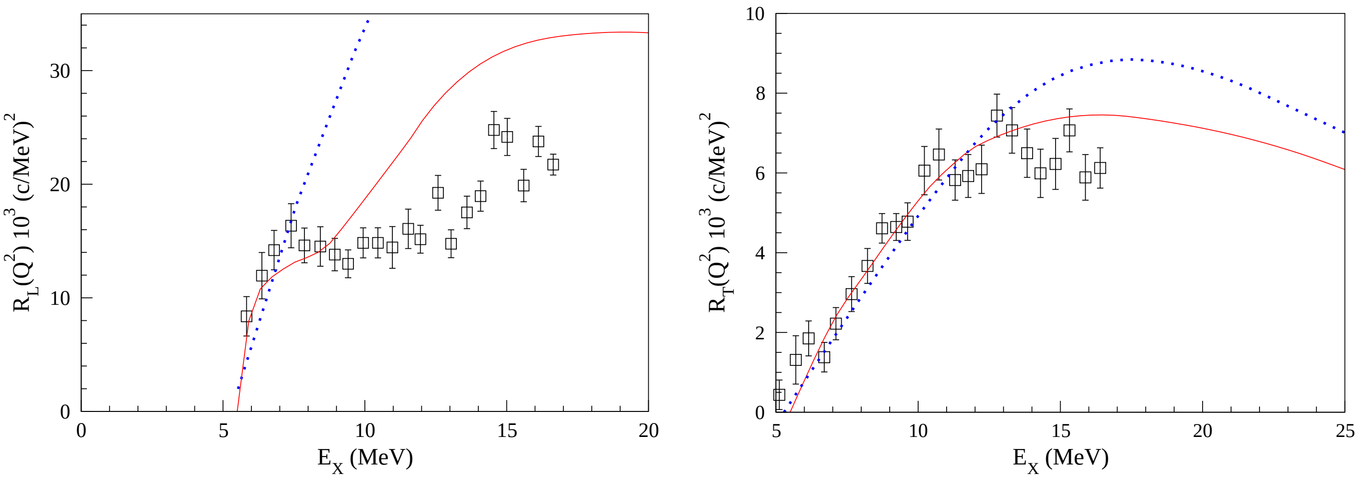
Calculations of both final states and em responses are highly non trivial numerical tasks, that require a very intensive computational effort!

Unpolarized Responses, R_L and R_T, vs the missing energy

$$E_X = \sqrt{(\omega + M_3)^2 - |\vec{q}|^2} - M_3 \simeq B_3 + E_{CM}^{fin}$$

E_{CM}^{fin} = three-nucleon energy in the final state.

three-momentum transfer $|\vec{q}| \sim 175\ MeV$



$\square$ $\equiv$ MIT data. (Retzlaff et al PRC **49**(1994) 1263)

Solid line: preliminary calculations with AV18 and Höhler Nucleon form factors (without Coulomb interaction) - For $J \leq 5/2$, full interaction in the final states with an incoming $|p\ ^2H\rangle$, for $5/2 < J \leq 13/2$ PWIA only

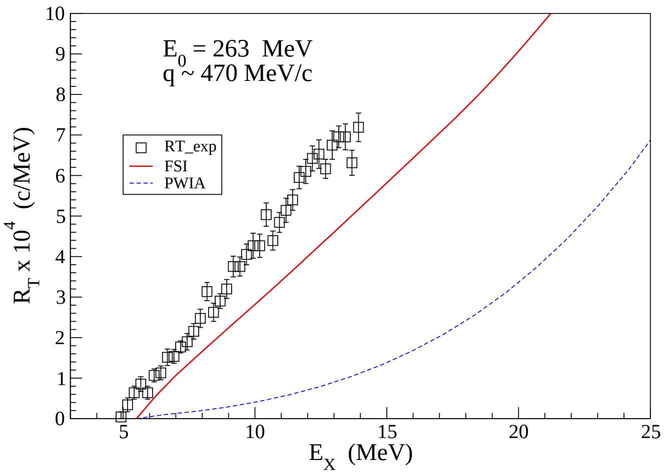
Dotted line: PWIA calculation, the spectator pair is interacting !

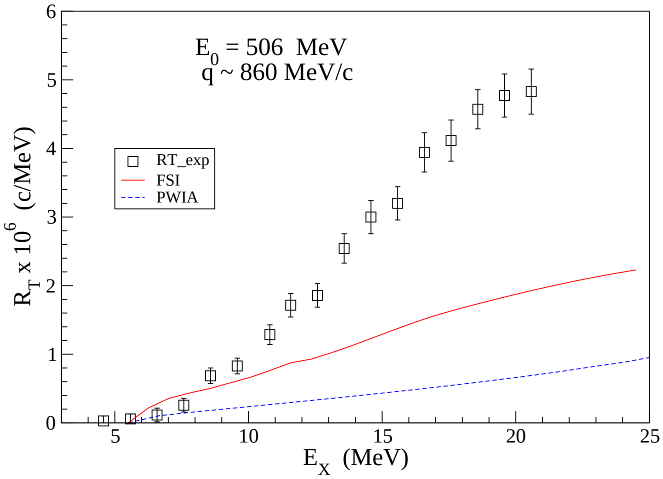
Unpolarized Transverse Response, R_T, vs the missing energy

$$E_X = \sqrt{(\omega + M_3)^2 - |\vec{q}|^2} - M_3 \simeq B_3 + E_{CM}^{fin}$$

E_{CM}^{fin} = three-nucleon energy in the final state.

three-momentum transfers: $|\vec{q}| = 470\ MeV$ and $|\vec{q}| = 860\ MeV$





□ ≡ New MIT data. (Hicks et al PRC **67**(2003) 064004)

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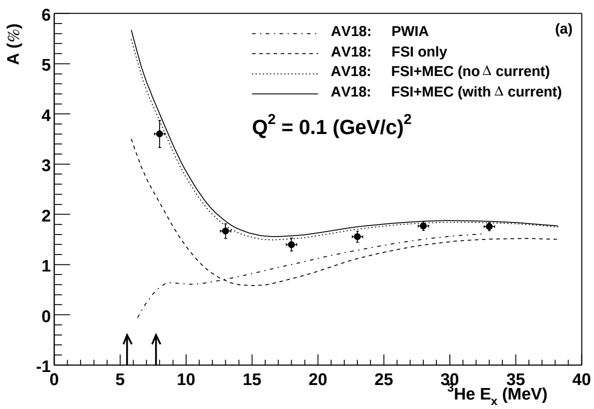
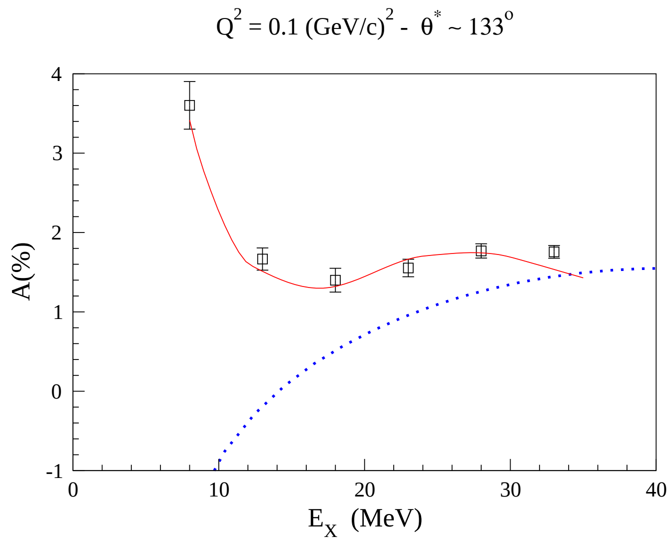
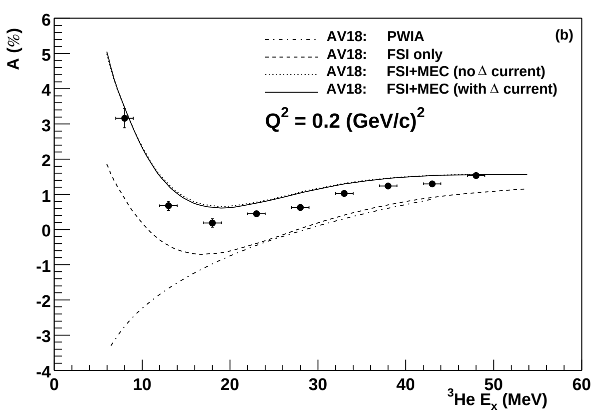
Transverse Asymmetry vs the missing energy

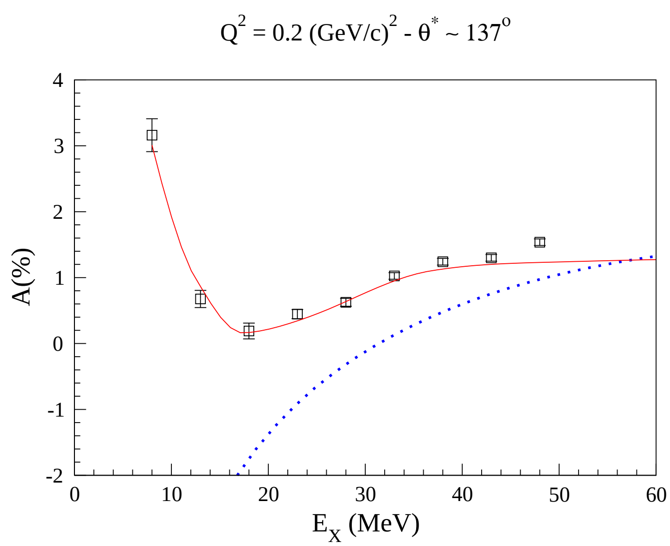
$$E_X = \sqrt{(\omega + M_3)^2 - |\vec{q}|^2} - M_3 \simeq B_3 + E_{CM}^{fin}$$

for two values of the four-momentum transfer $Q^2 = \omega^2 - |\vec{q}|^2$. (F. Xiong et al PRL **87**(2001) 242501)









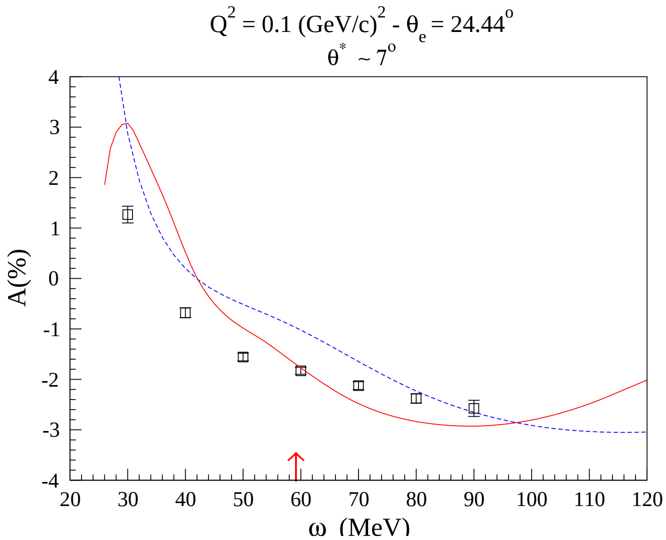
FSI: Bochüm

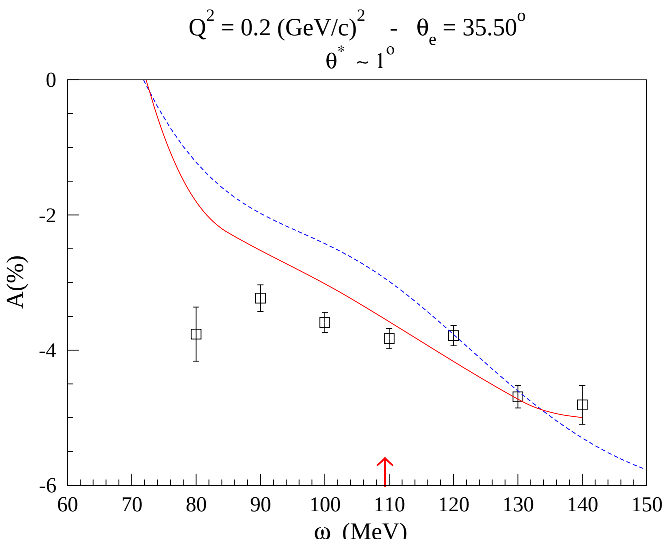
PWIA: Rome-Pisa

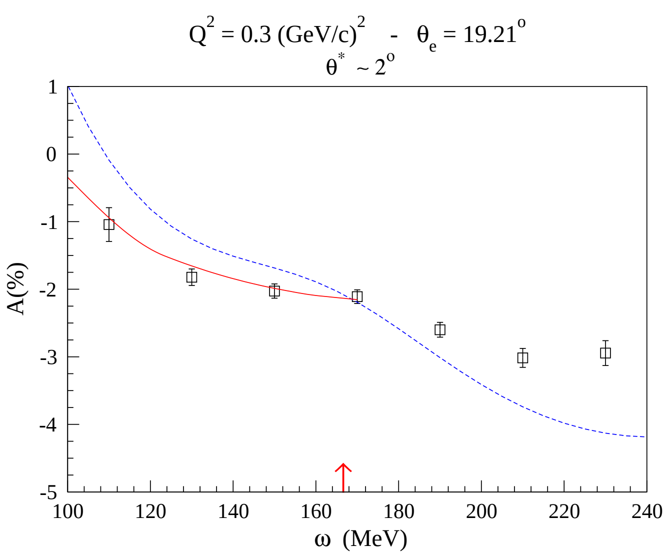
Solid line: FSI for the $\{p, d\}$ channel +PWIA

Dotted line: PWIA only

Transverse Asymmetry vs the energy transfer, ω ($Q^2 = \omega^2 - |\vec{q}|^2$). (W. Xu et al PRL **85**(2000) 2900)







□ ≡ TJLAB data.

Solid line: preliminary calculations with AV18 and Höhler Nucleon form factors (without Coulomb interaction) - FSI+PWIA

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At the quasielastic peak

$$Q^2 = 0.1\,(GeV/c)^2 \;\rightarrow 1 - PWIA/FSI \sim 40\%$$

$$Q^2 = 0.2\,(GeV/c)^2 \;\rightarrow 1 - PWIA/FSI \sim 20\%$$

$$Q^2 = 0.3\,(GeV/c)^2 \;\rightarrow 1 - PWIA/FSI \sim 5\%$$

In the coordinate space the 3-nucleon final state can be written as

$$\langle \mathbf{y}_1, \mathbf{x}_1 | j, j_z; TT_z, \pi, \epsilon_{int}; \beta \rangle = \frac{1}{\sqrt{3}} \sum_{L'X'} \sum_{j'_{23} S'_{23}} \sum_{\ell}$$

$$\mathcal{Y}^{j'_{23} S'_{23} \ell}_{L'X' j j_z T T_z}(\hat{x}_1, \hat{y}_1)\, \mathcal{J}^{j'_{23} S'_{23} \ell}_{L'X' j T}(|\mathbf{y}_1|, |\mathbf{x}_1|, \epsilon_{int}; \beta)$$

where $\beta \equiv \{\alpha, LX\}$ is the set of quantum numbers of the incoming wave

In order to give an insight on the structure of the three-nucleon wave functions with positive energy, let us introduce density functions for given J, parity and energy, viz

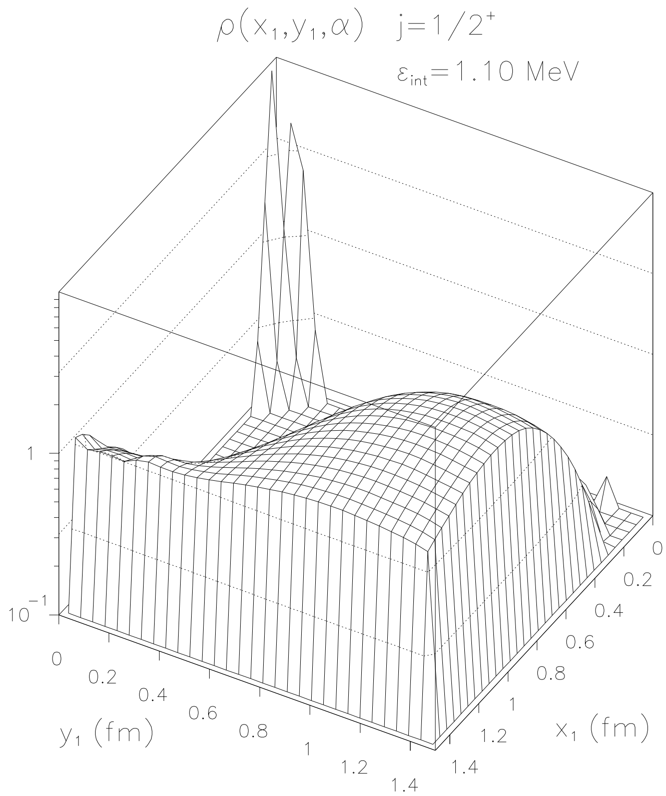
$$\rho(|\mathbf{y}_1|, |\mathbf{x}_1|, \alpha) =$$

$$= \sum_{LX} \sum_{L'X'} \sum_{j'_{23} S'_{23}} \sum_{\ell} \left\{ \left| \Re e \left[\mathcal{J}^{j'_{23} S'_{23} \ell}_{L'X' j T}(|\mathbf{y}_1|, |\mathbf{x}_1|, \epsilon_{int}; \beta) \right] \right|^2 + \right.$$

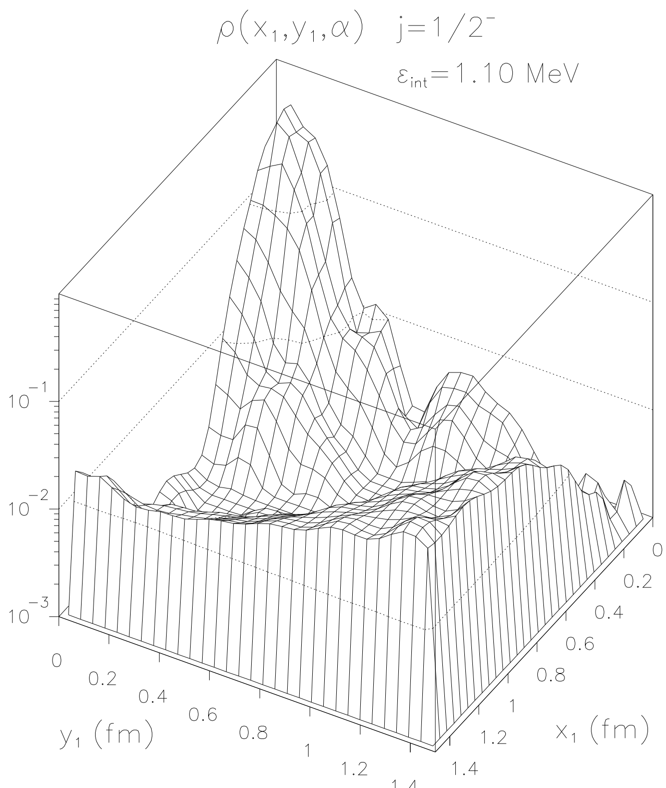
$$\left. + \left| \Im m \left[\mathcal{J}^{j'_{23} S'_{23} \ell}_{L'X' j T}(|\mathbf{y}_1|, |\mathbf{x}_1|, \epsilon_{int}; \beta) \right] \right|^2 \right\}$$

Such density functions yield information on the probability distribution of a nucleon in the final state, taking into account the caveat that the wave functions in the continuum are not square integrable.

Examples of the density function for a state with an asymptotic $p + d$ cluster are presented for $j = 1/2^+$ and $j = 1/2^-$, respectively, with $\epsilon_{int} = 1.1$ MeV.



The density function $\rho(|\mathbf{y}_1|, |\mathbf{x}_1|, \alpha)$ vs the Jacobi coordinates, $\{|\vec{x}_1|, |\vec{y}_1|\}$, for $J = 1/2$, $T = 1/2$, $\pi = +1$ and $\epsilon_{int} = 1.1$ MeV.



The density function $\rho(|\mathbf{y}_1|, |\mathbf{x}_1|, \alpha)$ vs the Jacobi coordinates, $\{|\vec{x}_1|, |\vec{y}_1|\}$ for $J = 1/2$, $T = 1/2$, $\pi = -1$ and $\epsilon_{int} = 1.1$ MeV.

Conclusions & Perspectives

- Dynamics $\rightarrow$ FSI
- Relativity $\rightarrow$ Kinematics, Poincaré covariance
- Electromagnetic current for interacting system (possibly coherent with the underlying dynamics and the Poincaré covariance)