A NEW REALISTIC MANY-BODY APPROACH FOR THE DESCRIPTION OF HIGH-ENERGY SCATTERING PROCESSES OFF COMPLEX NUCLEI

M. Alvioli, C. Ciofi degli Atti, H. Morita

I Realistic Wave Functions for Complex Nuclei

II $A(e, e'p)X$ scattering processes

III Total n-A cross section

I.1 - *The Interacting Many-Hadron Problem*

- the non-relativistic Schrödinger equation:

$$\hat{\mathbf{H}}\,\Psi_n \;=\; E_n\,\Psi_n\,, \quad \textit{with}: \quad \hat{\mathbf{H}} \;=\; \hat{T}+\hat{V} \;=\; -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{i<j}\hat{v}_{ij}$$

where the *realistic potential* appears:

$$\hat{v}_{ij} \;=\; \sum_n v^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

$$\hat{\mathcal{O}}_{ij}^{(n)} \;=\; \Big[1\,,\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\,,\,\hat{S}_{ij}\,,\,(\boldsymbol{L}\cdot\boldsymbol{S})_{ij}\,,\,...\Big]\otimes\big[1\,,\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\big]$$

- the same operatorial dependence is cast onto ψ_o:

$$\Psi_o \;\rightarrow\; \psi_o \;=\; \hat{\mathbf{F}}\,\phi_o \;=\; \hat{F}\,det\left\{\varphi_i(\boldsymbol{x}_j)\right\}\,/\sqrt{A!}$$

where φ is a single-particle mean-field wave function and

$$\hat{\mathbf{F}} \;=\; \hat{S}\prod_{i<j}\hat{f}_{ij} \;=\; \hat{S}\prod_{i<j}\sum_n f^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

I.2 - *Cluster Expansion*

- The expansion acts on the numerator and denominator of an operator expectation value ($F^2 = \prod f^2 \simeq \sum(1+\eta)$):

$$\frac{\langle\psi_o|\,\hat{\mathcal{O}}\,|\psi_o\rangle}{\langle\psi_o|\,\psi_o\rangle} = \frac{\langle\phi_o|\,F^\dagger\,\hat{\mathcal{O}}\,F\,|\phi_o\rangle}{\langle\phi_o|\,F^\dagger\,F\,|\phi_o\rangle} = \frac{\langle\phi_o|\,F^2\,\hat{\mathcal{O}}\,|\phi_o\rangle}{\langle\phi_o|\,F^2\,|\phi_o\rangle} =$$

$$\simeq \frac{\langle\phi_o|\,(1+\eta)\,\hat{\mathcal{O}}\,|\phi_o\rangle}{1+\langle\phi_o|\,\eta\,|\phi_o\rangle} =$$

$$\simeq \left[\langle\phi_o|\,\hat{\mathcal{O}}\,|\phi_o\rangle + \langle\phi_o|\sum_{i>j}\eta_{ij}\,\hat{\mathcal{O}}\,|\phi_o\rangle\right]\cdot\left[1 - \langle\phi_o|\sum_{i>j}\eta_{ij}\,|\phi_o\rangle\right] =$$

$$\simeq \langle\phi_o|\,\hat{\mathcal{O}}\,|\phi_o\rangle + \langle\phi_o|\sum_{i>j}\eta_{ij}\,\hat{\mathcal{O}}\,|\phi_o\rangle - \langle\phi_o|\,\hat{\mathcal{O}}\,|\phi_o\rangle\,\langle\phi_o|\sum_{i>j}\eta_{ij}\,|\phi_o\rangle\,,$$

in which we dropped terms $\propto\ \mathcal{O}(\eta^2) \equiv \mathcal{O}(f^4)$ ...

... obtaining: $\langle\hat{\mathcal{O}}\rangle = \mathcal{O}_0 + \mathcal{O}_1 + \mathcal{O}_2 + \ldots,$ with:

$$\mathcal{O}_0 = \langle\hat{\mathcal{O}}\rangle \equiv \langle\phi_o|\hat{\mathcal{O}}|\phi_o\rangle\,,$$

$$\mathcal{O}_1 = \langle\sum_{ij}\eta_{ij}\,\hat{\mathcal{O}}\rangle - \mathcal{O}_0\,\langle\sum_{ij}\,\eta_{ij}\rangle \equiv \langle\sum_{ij}\eta_{ij}\,\hat{\mathcal{O}}\rangle_{Linked}\,;$$

where at each order n of the expansion only powers of

$$\eta^{n/2} \equiv f^n$$

appear, and we use the notation:

$$\eta_{ij}\,\hat{\mathcal{O}} \equiv f_{ij}\,\hat{\mathcal{O}}\,f_{ij} - \hat{\mathcal{O}}\,;$$

$$\eta_{ij}\,\eta_{kl}\,\hat{\mathcal{O}} \equiv f_{ij}\,f_{kl}\,\hat{\mathcal{O}}\,f_{kl}\,f_{ij} - f_{ij}\,\hat{\mathcal{O}}\,f_{ij} - f_{kl}\,\hat{\mathcal{O}}\,f_{kl} + \hat{\mathcal{O}}\,.$$

I.3 - *Energy of the Ground State*

- the solution for ψ_o can be found variationally; the variational parameters are in the correlation functions and in the single particle wave function
- we calculated E_o by means of the **1**− and **2**−body *densities*:

$$\mathbf{E_o} = -\frac{\hbar^2}{2\,m}\int d\boldsymbol{r}\left[\hat{\nabla}^2\rho^{(1)}(\boldsymbol{r},\boldsymbol{r}')\right]_{\boldsymbol{r}=\boldsymbol{r}'} + \sum_n \int d\boldsymbol{r}_1 d\boldsymbol{r}_2\,\hat{v}^{(n)}\rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

$$\longrightarrow\ \rho^{(1)}(\boldsymbol{r},\boldsymbol{r}') = A\int\prod_{j=2}^{A} d\boldsymbol{r}_j\,\Psi_o^\dagger(\boldsymbol{r},\boldsymbol{r}_2\ldots,\boldsymbol{r}_A)\,\Psi_o(\boldsymbol{r}',\boldsymbol{r}_2\ldots,\boldsymbol{r}_A)$$

$$\longrightarrow\ \rho^{(2)}_{(n)}(1,2) = \frac{A(A-1)}{2}\int\prod_{j=3}^{A} d\boldsymbol{r}_j\,\Psi_o^\dagger(\boldsymbol{r}_1\ldots,\boldsymbol{r}_A)\,\hat{O}^{(n)}_{12}\,\Psi_o(\boldsymbol{r}_1\ldots,\boldsymbol{r}_A)$$

- we used the ***cluster expansion*** technique for the ground state wave function, ψ_o, in order to evaluate E_o.

• at first order of the $\eta-$expansion, the **full correlated** *one-body mixed* density matrix expression is as follows:

$$\rho^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1) = \rho_o^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1) + \rho_H^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1) + \rho_S^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1), \quad (1)$$

with

$$\rho_H^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1) = \int d\boldsymbol{r}_2 \Big[H_D(r_{12}, r_{1'2})\, \rho_o^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1)\, \rho_o(\boldsymbol{r}_2) - H_E(r_{12}, r_{1'2})\, \rho_o^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}_2)\, \rho_o^{(1)}(\boldsymbol{r}_2, \boldsymbol{r}'_1) \Big]$$

$$\rho_S^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}'_1) = -\int d\boldsymbol{r}_2 d\boldsymbol{r}_3 \rho_o^{(1)}(\boldsymbol{r}_1, \boldsymbol{r}_2) \Big[H_D(r_{23}) \rho_o^{(1)}(\boldsymbol{r}_2, \boldsymbol{r}'_1) \rho_o(\boldsymbol{r}_3) - H_E(r_{23}) \rho_o^{(1)}(\boldsymbol{r}_2, \boldsymbol{r}_3) \rho_o^{(1)}(\boldsymbol{r}_3, \boldsymbol{r}'_1) \Big]$$

and the functions H_D and H_E are defined as:

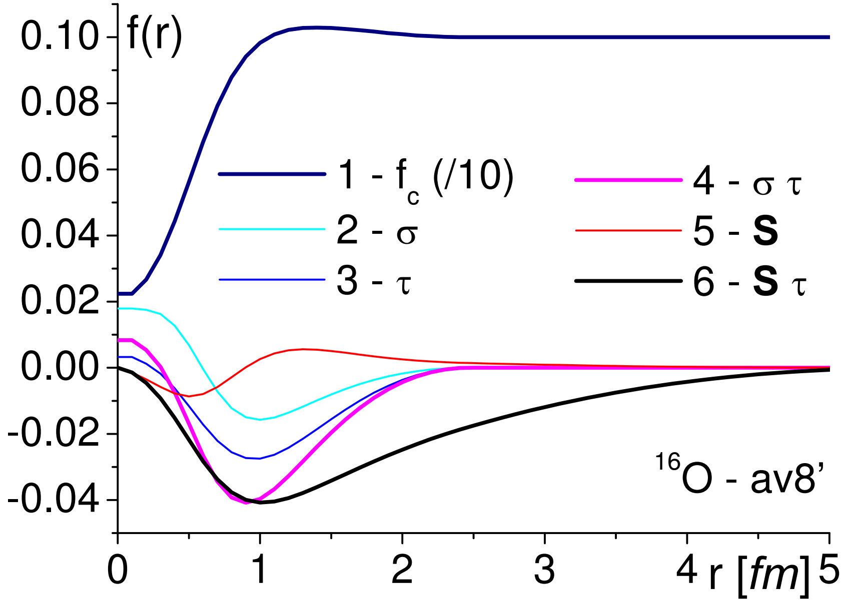
$$H_{D(E)}(r_{ij}, r_{kl}) = \sum_{p,q=1}^{6} f^{(p)}(r_{ij})\, f^{(q)}(r_{kl})\, C_{D(E)}^{(p,q)}(r_{ij}, r_{kl}) - C_{D(E)}^{(1,1)}(r_{ij}, r_{kl}) \,.$$

with $C_{D(E)}^{(p,q)}$ proper coefficients coming from spin-isospin traces;

• a similar, complicated expression holds for the *two-body* density matrix.

I.4 - *Energy Results*

- **ingredients** for ψ_o: correlation $f^{(n)}$ and φ; $f^{(n)}$, $n = 1, ..., 6$ are:



FHNC
variational calculation
(Pisa-Lecce)
Phys. Rev. ***C61***
(2000) 044302

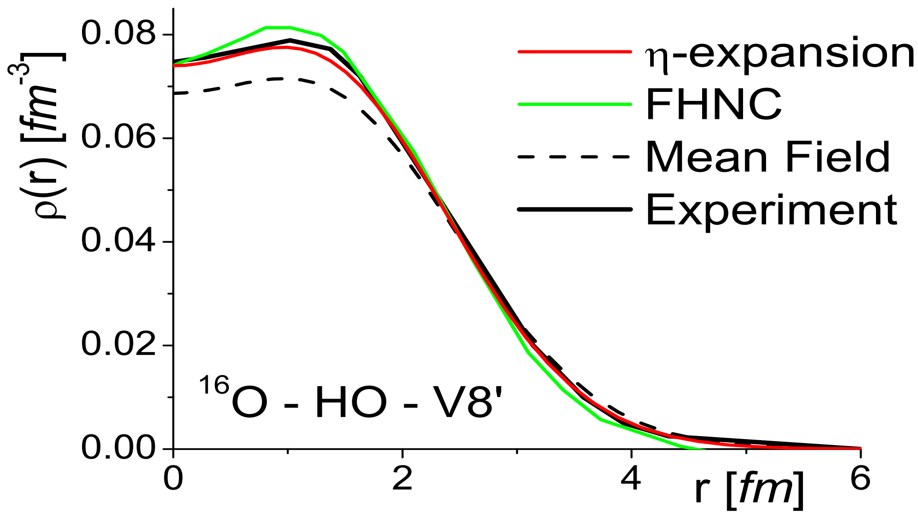
^{16}O benchmark calculation - truncated $V8'$ interaction

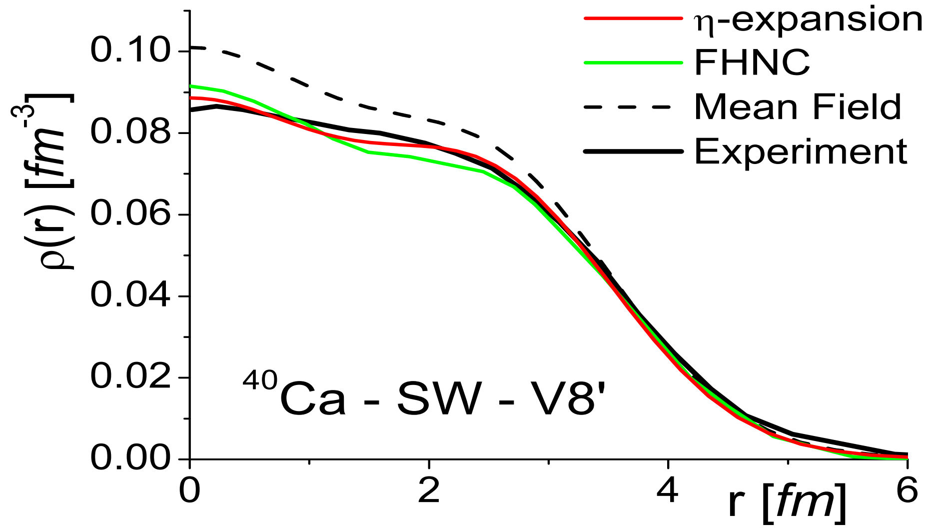
	V_c	V_σ	V_τ	$V_{\sigma\tau}$	V_S	$V_{S\tau}$	**V**	**T**	$\mathbf{E_o}$	$\mathbf{E_o/A}$
$\eta-exp$	0.64	-35.35	-10.1	-172.8	-0.03	-172.7	-390.37	323.50	-66.87	-4.18
FHNC	0.69	-40.1	-10.6	-180.0	-0.07	-160.3	-390.30	325.18	-65.12	-4.07

(*M.Alvioli, C.Ciofi degli Atti, H.Morita*; `nucl-th/0506054`)

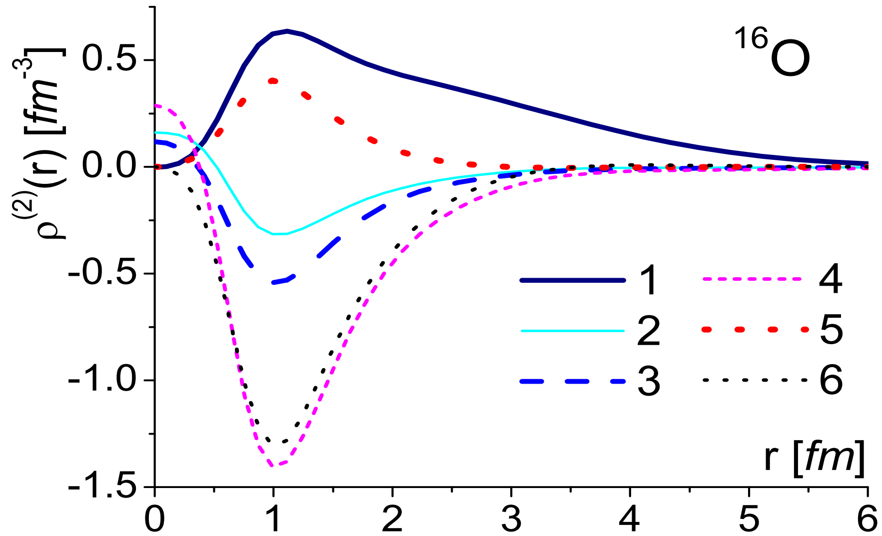
I.5 - *Results for the Densities*

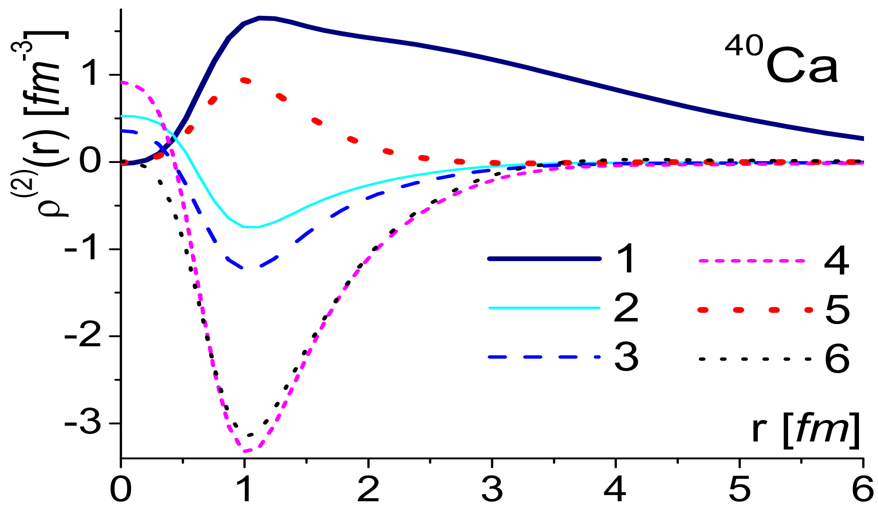
- the charge densities $\rho^{(1)}(\boldsymbol{r},\boldsymbol{r'})|_{\boldsymbol{r'}=\boldsymbol{r}}$ for ^{16}O and ^{40}Ca:



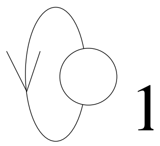


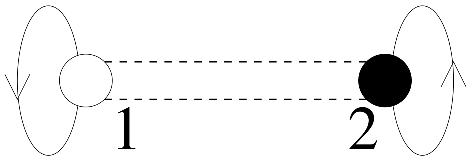
- the two-body densities $\rho^{(2)}_{(n)}(\boldsymbol{r}) = \int d\boldsymbol{R}\, \rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2)$

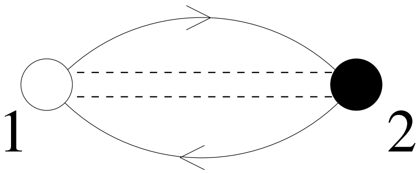


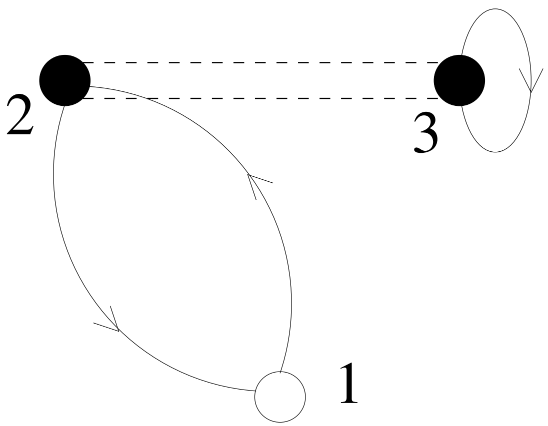


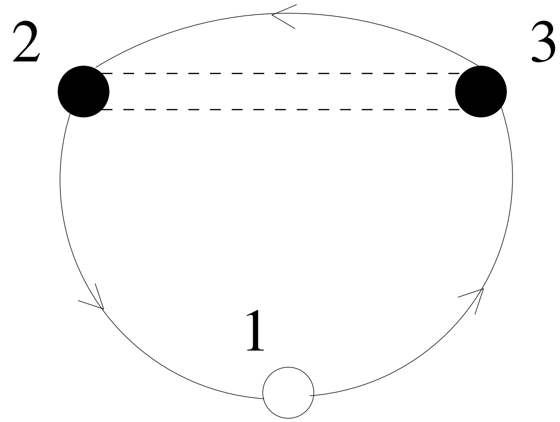
Densities: Diagrams at 1−st order of the η−expansion

Mean Field ⟶ 



⟵ *Hole* ⟶


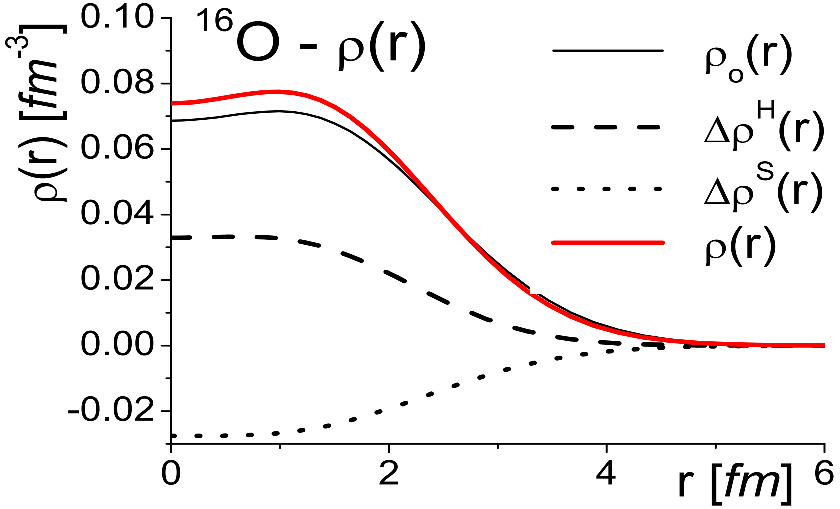


⟵ *Spectator* ⟶


the **unlinked** diagrams cancel out at each order

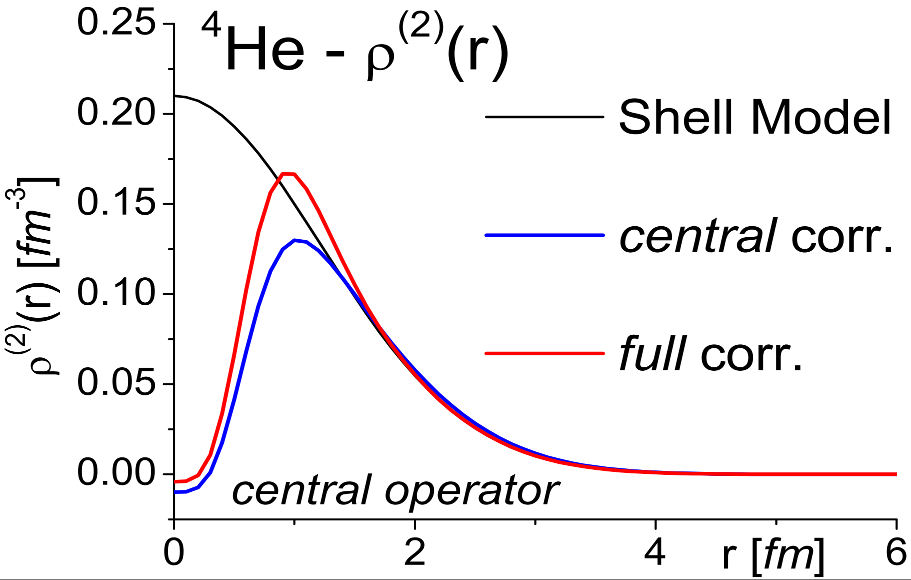
Densities: Effect of Correlations

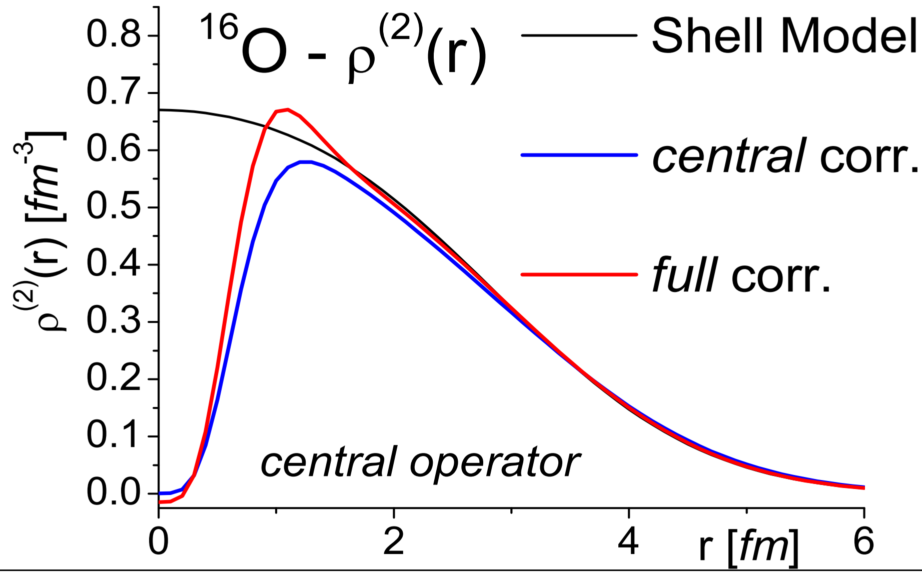
- charge density $\rho^{(1)}(\boldsymbol{r},\boldsymbol{r'})|_{\boldsymbol{r'}=\boldsymbol{r}}$ for ^{16}O



$$\rho_O(r) \longrightarrow \textit{Mean Field}$$
$$\Delta\rho^H(r) \longrightarrow \textit{Hole contribution}$$
$$\Delta\rho^S(r) \longrightarrow \textit{Spectator contribution}$$
$$\rho(r) \longrightarrow \textit{Full Correlations}$$

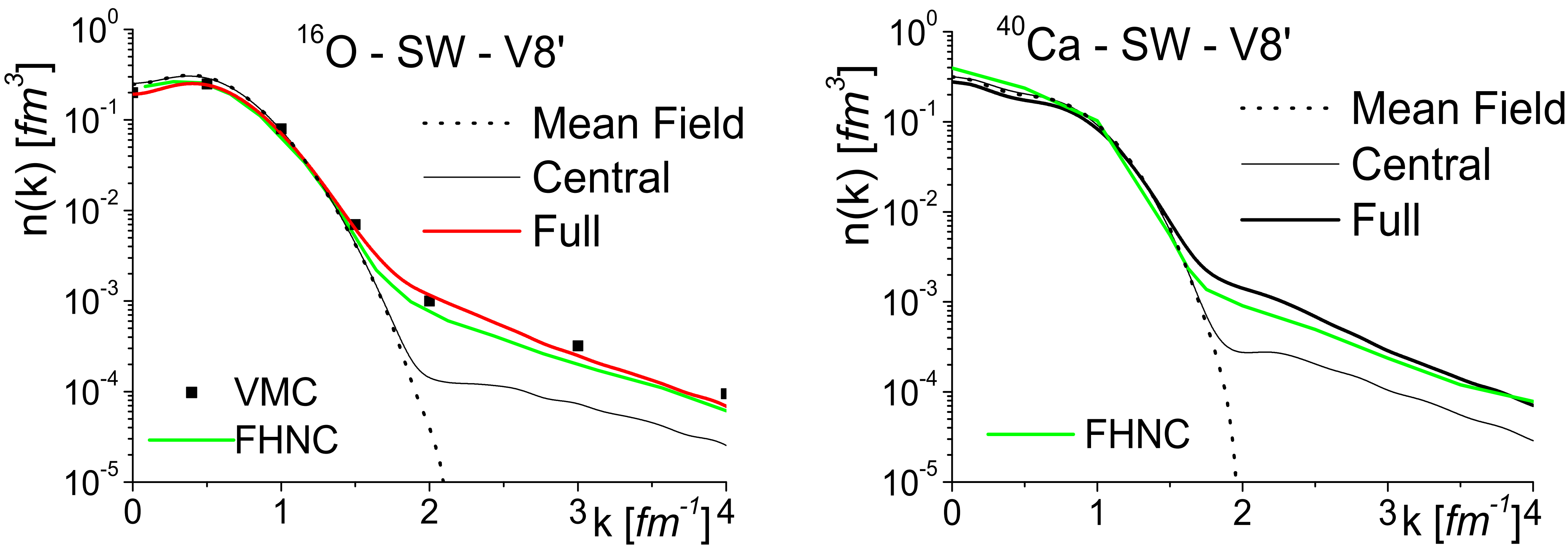
- the two-body densities $\rho^{(2)}_{(n)}(\boldsymbol{r}) = \int d\boldsymbol{R}\,\rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2)$





I.6 - *Results for the Momentum Distributions*

- by means of the obtained wave functions we can calculate the nucleon momentum distribution



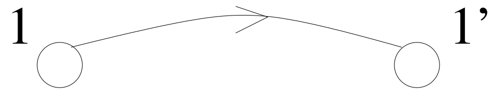
<u>*FHNC*</u>: *Fabrocini, de Saavedra, Co', Phys. Rev.* ***C61****, (2000) 044302*

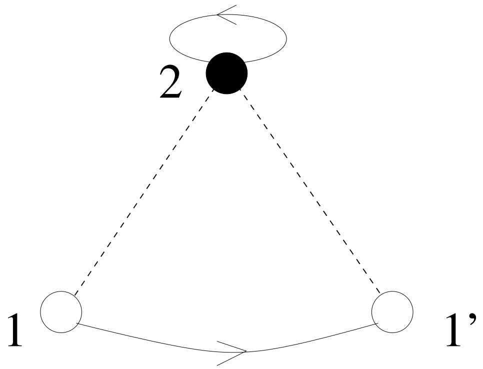
<u>*VMC*</u>: *Pieper, Wiringa, Pandharipande, Phys. Rev.* ***C46****, (1992) 1741*

<u>$\eta - exp.$</u>: *M.Alvioli, C.Ciofi degli Atti, H.Morita*; `nucl-th/0506054`

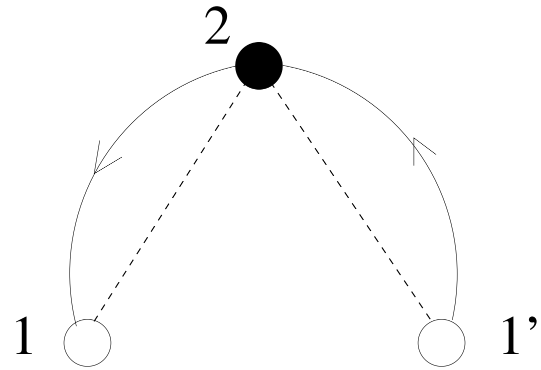
$\mathbf{n(k)}$: *Diagrams at 1−st order of the η−expansion*

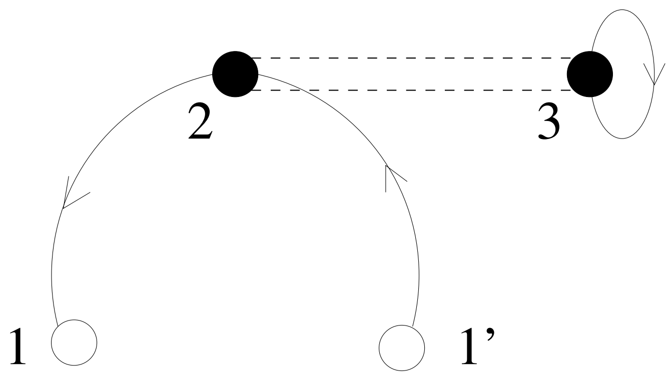
Mean Field ⟶



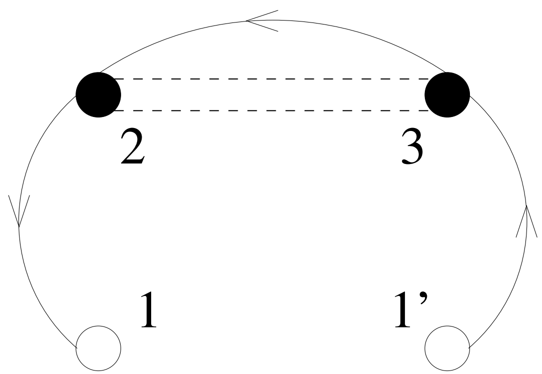


⟵ *Hole* ⟶

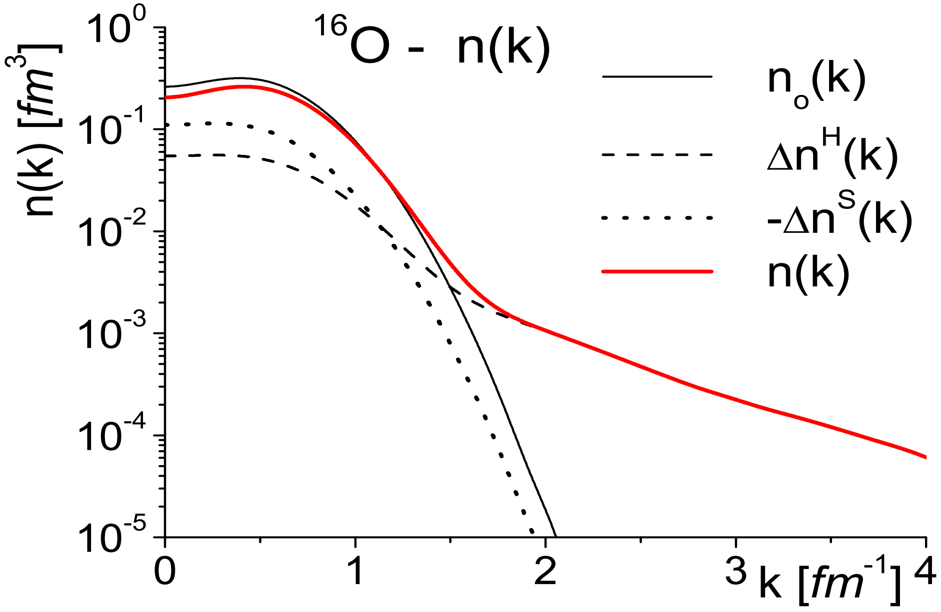




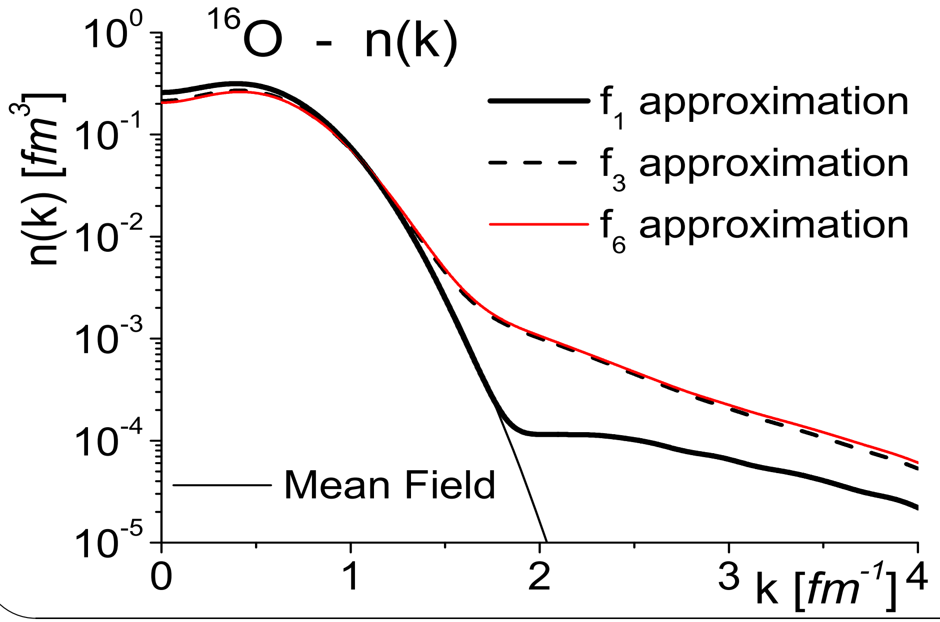
⟵ *Spectator* ⟶



$\mathbf{n(k)}$: *Effect of Correlations*



$$n_O(k) \longrightarrow \textit{Mean Field}$$
$$\Delta n^H(k) \longrightarrow \textit{Hole contribution}$$
$$\Delta n^S(k) \longrightarrow \textit{Spectator contribution}$$
$$n(k) \longrightarrow \textit{Full Correlations}$$



$$f_1 \longrightarrow \textit{Central Correlation}$$
$$f_3 \longrightarrow f^{(n)} \textit{ with } n = 1, 4, 6 \textit{ only}$$
$$f_6 \longrightarrow \textit{Full Correlations}$$

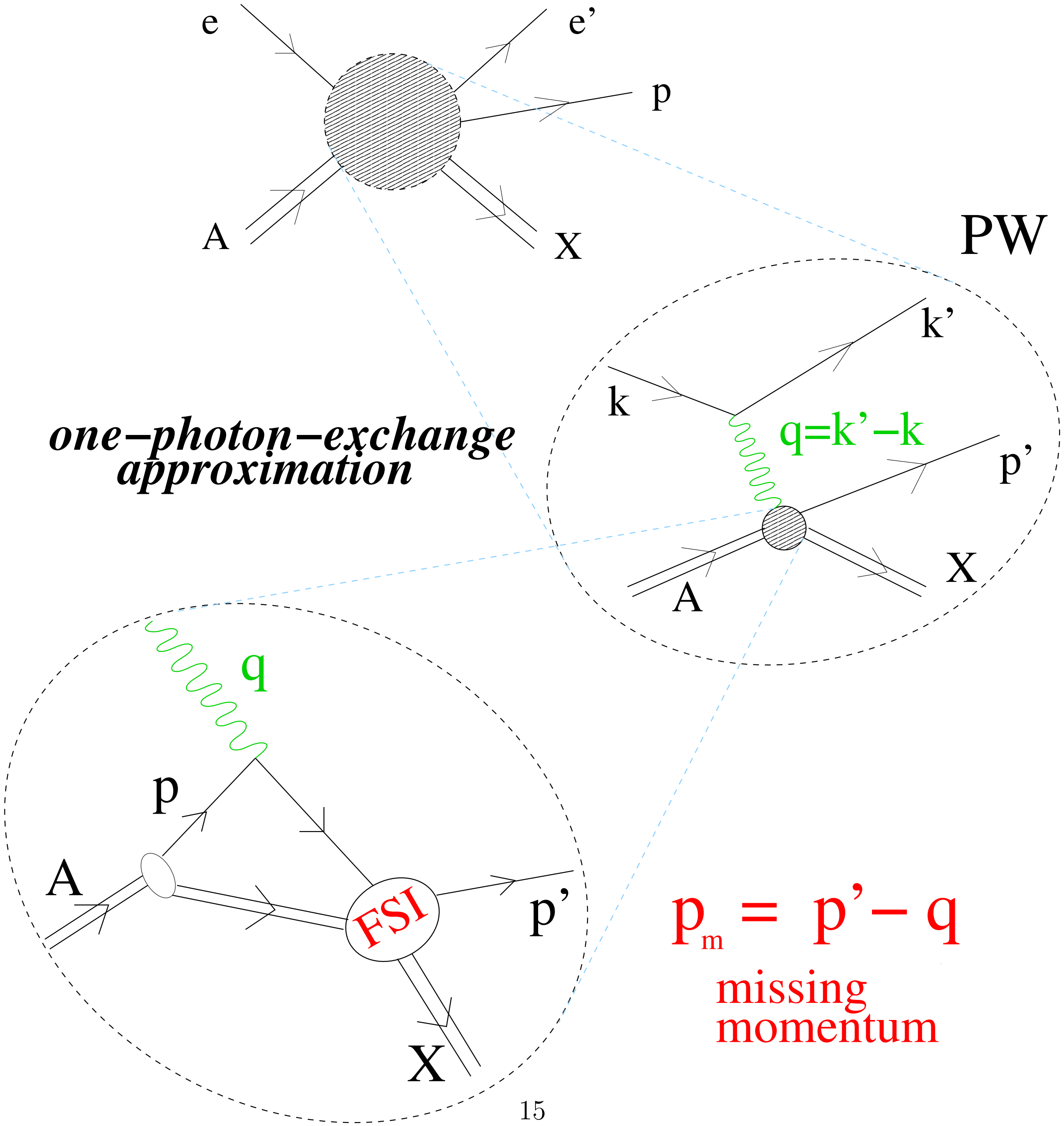
*Comments and conclusions **I***

- within a new cluster expansion we have obtained reasonable E_o, $\rho(r)$ and $n(k)$, qualitatively and quantitatively comparable with FHNC/SOC and VMC calculations, which are very complex to be used in actual calculations

- the calculation of each value of E_o *at first order of* $\eta-$*expansion* requires about *15 hours* on 20 CPUs. We are trying to evaluate the relevant *second order* terms

- we have at disposal a realistic ψ_o, which explains the quantities we have illustrated, to be used in high energy scattering processes calculations

II.1 - *The semi-inclusive* $A(e, e'p)X$ *process*



- *factorized* cross section:

$$\frac{d\sigma}{dQ^2\, d\nu\, d\boldsymbol{p'}} = K\, \sigma_{ep}\, n_D(\boldsymbol{p}_m)$$

- the FSI-distorted momentum distribution $n_D(\boldsymbol{p}_m)$ can be calculated by:

$$n_D(\boldsymbol{p}_m) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r}_1 d\boldsymbol{r}'_1\, e^{i\,\boldsymbol{p}_m\cdot(\boldsymbol{r}_1 - \boldsymbol{r}'_1)} \times$$

$$\times \underbrace{\int \prod_{j=2}^{A} \psi_o^\dagger(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)\, \hat{S}^\dagger\, \hat{S}\, \psi_o(\boldsymbol{r}'_1, \boldsymbol{r}_2 ..., \boldsymbol{r}_A)}_{\rho_D(\boldsymbol{r}, \boldsymbol{r}')}$$

- we first calculate the distorted density within the $\eta-$expansion:

$$\rho_D(\boldsymbol{r}, \boldsymbol{r}') = \frac{\langle\psi_o|\, \hat{S}_G^\dagger\; \hat{\rho}^{(1)}(\boldsymbol{r}, \boldsymbol{r}')\; \hat{S}_G\, |\psi_o\rangle}{\langle\psi_o|\, \hat{S}_G^\dagger\, \hat{S}_G\, |\psi_o\rangle} \quad \Longrightarrow \quad n_D(\boldsymbol{p}_m)$$

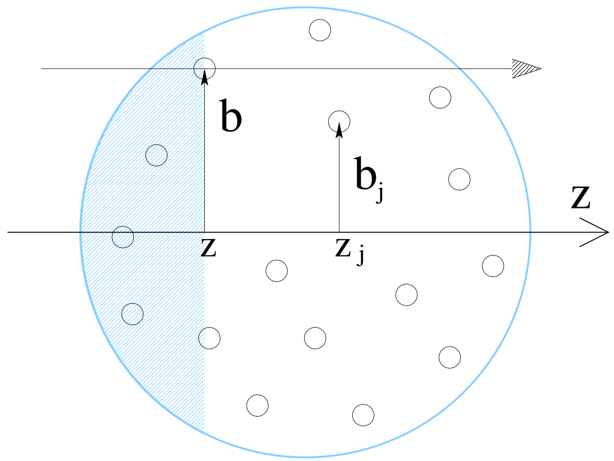
II.2 - *Final state interaction: Glauber*

- $\hat{\mathbf{S}}$ describes the interaction between the hit proton and the residual system
- $\hat{\mathbf{S}} \rightarrow \mathbf{S_G}$ calculated within the Glauber *multiple scattering* theory
- the interaction with *each* nucleon is parameterized by

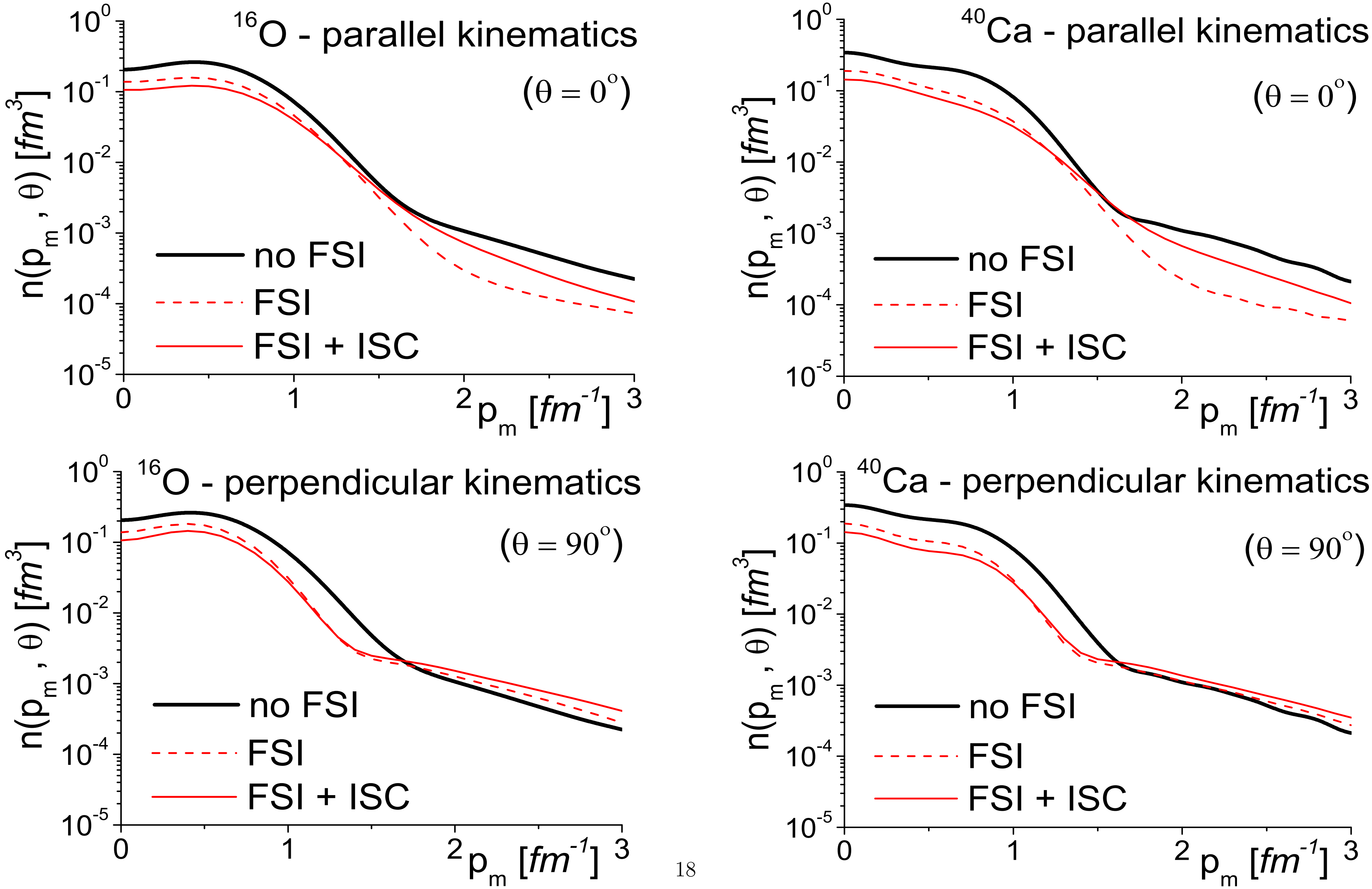
$$S_G(\boldsymbol{b}_1, ..., \boldsymbol{b}_A) = \prod_{j=2} \left[1 - \theta(z_j - z_1)\, \Gamma(\boldsymbol{b}_1 - \boldsymbol{b}_j)\right] =$$

$$= \prod_{j=2} \left[1 - \theta(z_j - z_1) \frac{\sigma_{pN}^{tot}}{4\pi B^2} \left(1 - i\,\alpha_{pN}\right) e^{-(\boldsymbol{b}_1 - \boldsymbol{b}_j)^2/2\,B}\right]$$

which depends upon the impact parameter $\boldsymbol{b} - \boldsymbol{b}_j$ and upon the parameters $\sigma_{pN}^{tot}(p)$, $\alpha_{pN}(p)$ and $B(p)$ ($p =$ proton momentum)

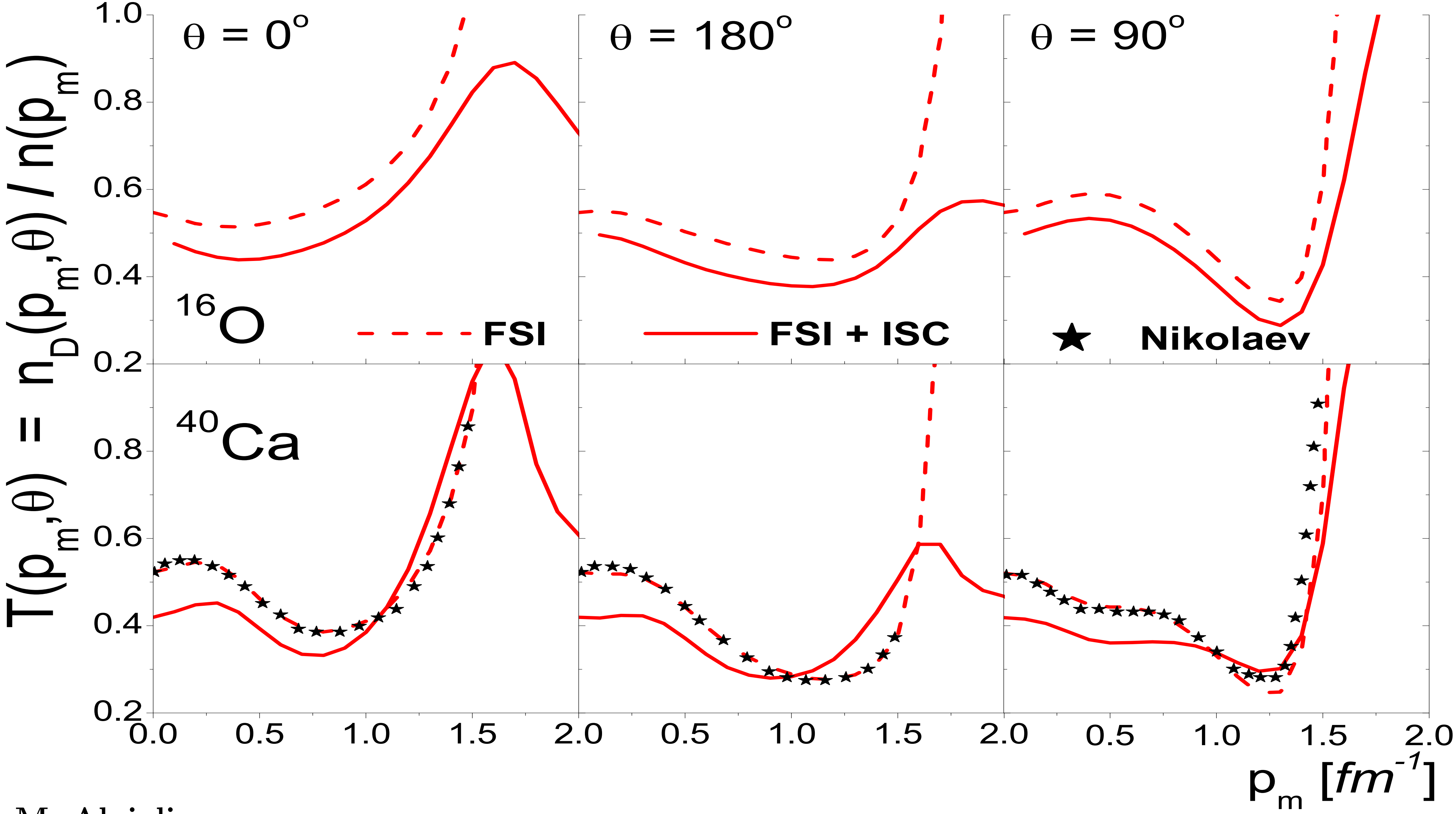


II.3 - *Results for* $n_D(\boldsymbol{p}_m)$

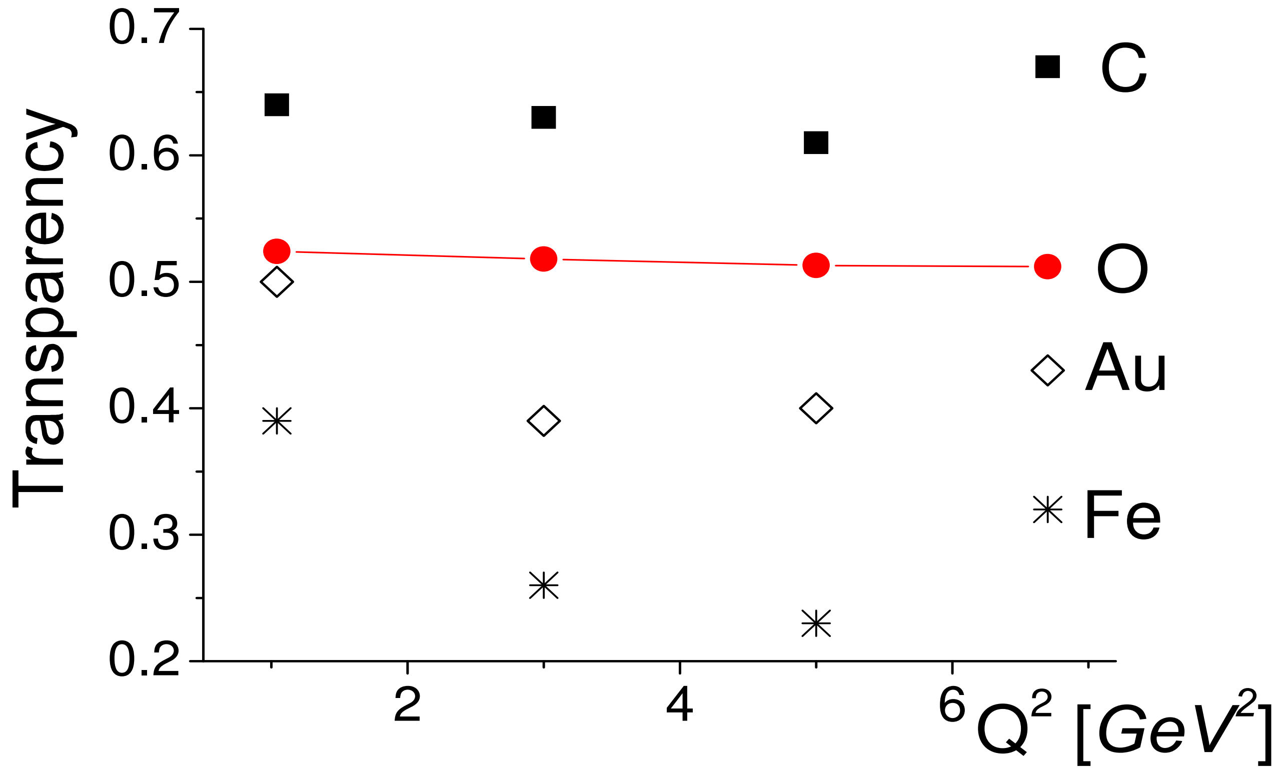


II.4 - *Non-Integrated Nuclear Transparency*

- the *nuclear transparency* allows one to compare FSI and PWIA results:



II.5 - *Integrated Nuclear Transparency*

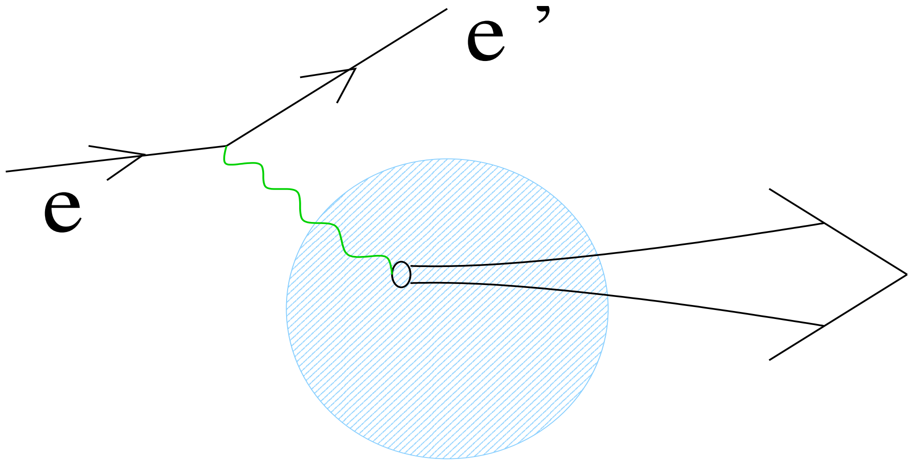


$$\mathbf{T} = \frac{\int \mathbf{dp_m d}\theta \;\; \mathbf{n_D}(\mathbf{p_m}, \theta)}{\int \mathbf{dp_m d}\theta \;\; \mathbf{n}(\mathbf{p_m}, \theta)} \equiv \frac{\sigma_{\mathbf{FSI}}}{\sigma_{\mathbf{PWIA}}}$$

Glauber $\longrightarrow$ *flat* Q^2 *dependence*

II.6 - *Finite Formation Time & Color Transparency*

- the *color transparency* phenomenon is predicted by non-perturabtive QCD calculations: at sufficiently high transferred Q^2, $\gamma^\star$ hits a nucleon in a



colorless, *pointlike* configuration, which is supposed to have *no FSI* with $A-1$ until it evolves into a physical nucleon

- we adopted the approach introduced in

Braun, Ciofi degli Atti, Treleani, *Phys. Rev.* **C62** *(2001)*

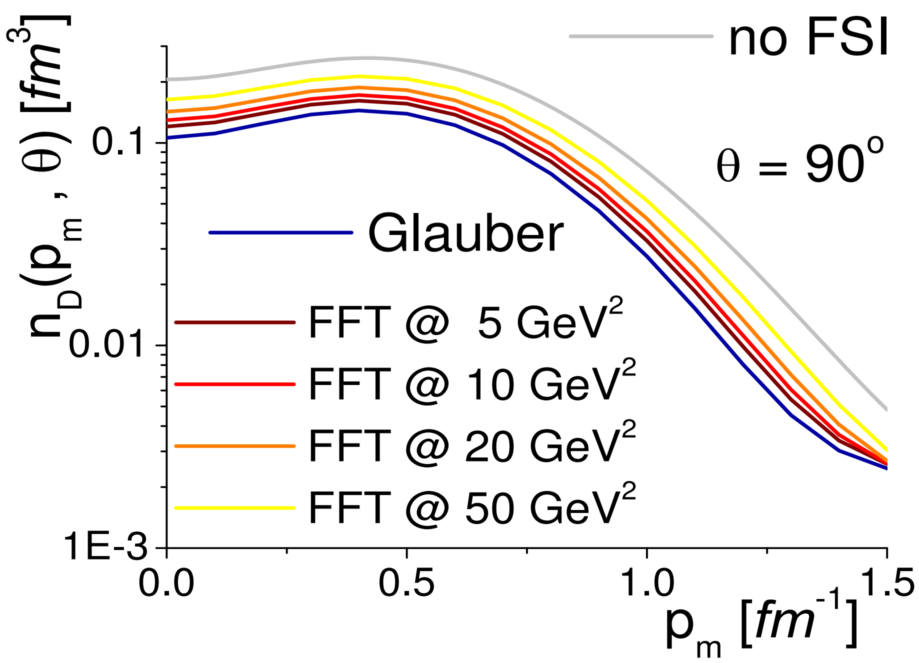
for the nucleon evolution towards its asymptotic state.

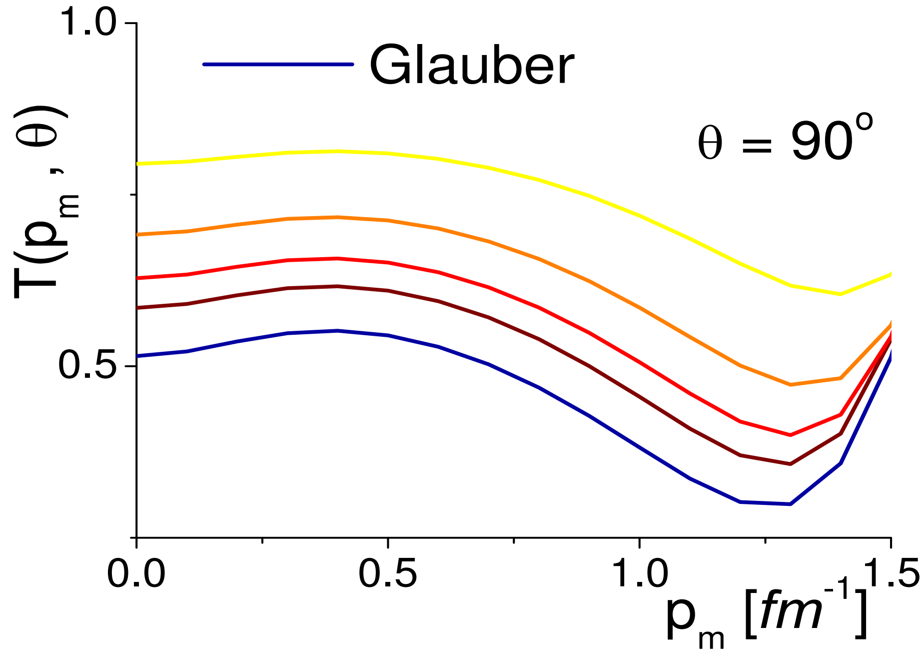
The **FFT** approach consists in the implementation of the operator $\neq$ Glauber:

$$J(\boldsymbol{b}, \boldsymbol{b}_j) \;=\; 1 \;-\; \theta(z_j - z)\; \left(1 \,-\, e^{-x\,\frac{mM^{\star}}{Q^2}\,z}\right)\; \Gamma(\boldsymbol{b} - \boldsymbol{b_j})$$

with $x \equiv x_{Bj} = Q^2/2m\nu = 1$; one has:

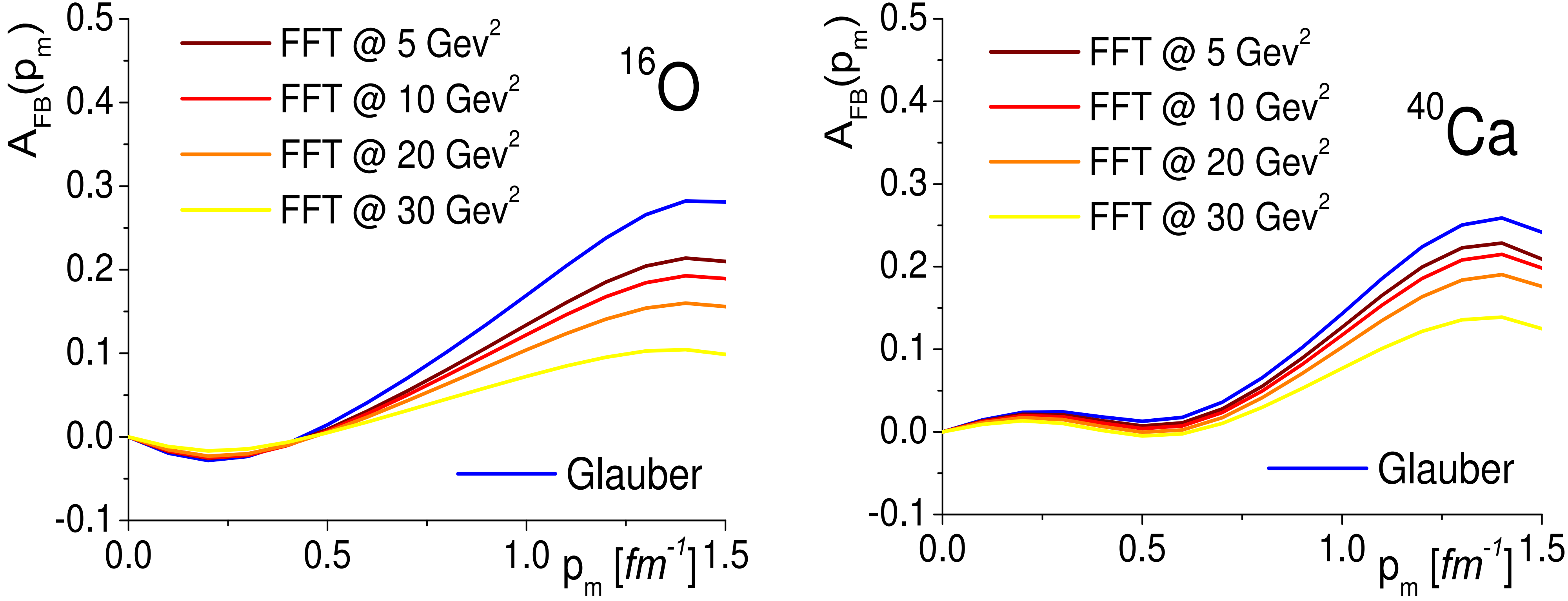
$$Q^2 \;>>\; m\,M^{\star}\,x \;\longrightarrow\; NO\;FSI$$





Forward-Backward asymmetry

$$A_{FB} = \frac{\sigma(p_m, \theta = 0^o) - \sigma(p_m, \theta = 180^o)}{\sigma(p_m, \theta = 0^o) + \sigma(p_m, \theta = 180^o)}$$



III - *n-A total cross section*

- We start from the *optical theorem*:

$$\sigma_{tot} = \frac{4\pi}{k} Im \left[F_{00}(0) \right]$$

- with the elastic scattering amplitude in the eikonal approximation:

$$F_{00}(0) = \frac{ik}{2\pi} \int e^{i\mathbf{q}\cdot\mathbf{b_n}} \langle\Psi_0|\{1-\hat{S}_G\}|\Psi_0\rangle = \frac{ik}{2\pi} \int d^2 b_n e^{i\mathbf{q}\cdot\mathbf{b_n}} \left[1 - \mathbf{e}^{\mathbf{i}\,\chi_{\mathbf{opt}}(\mathbf{b_n})} \right]$$

in which the optical *phase shift* is:

$$\mathbf{e}^{\mathbf{i}\,\chi_{\mathbf{opt}}(\boldsymbol{b}_{\mathbf{n}})} = \int \prod_{j=1}^{A} d\boldsymbol{r}_j\, G(\boldsymbol{b}_n, \boldsymbol{s}_j) \, |\Psi_0(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)|^2 \delta\left(\frac{1}{A} \sum \mathbf{r}_j \right)$$

- using the wave function of section *I*, we obtained the cross-section at

$\longrightarrow$ zeroth-order (*i.e. mean field*) expansion

$\longrightarrow$ first-order $\eta-$expansion (*i.e.* with *correlations*)

The expansion in terms of density distributions reads:

$$|\psi(\boldsymbol{r}_1,...,\boldsymbol{r}_A)|^2\,\delta\left(\frac{1}{A}\sum \mathbf{r}_j\right) = \prod_{j=1}^{A}\rho(\boldsymbol{r}_j) + \sum_{i<j=1}^{A}\boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{i},\boldsymbol{r}_\mathbf{j})\prod_{k\neq(il)}^{A}\rho(\boldsymbol{r}_k) + ...$$

where the *two-body contraction* Δ is

$$\boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{1},\boldsymbol{r}_\mathbf{2}) = \left[\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2) - \rho^{(1)}(\boldsymbol{r}_1)\,\rho^{(1)}(\boldsymbol{r}_2)\right];$$

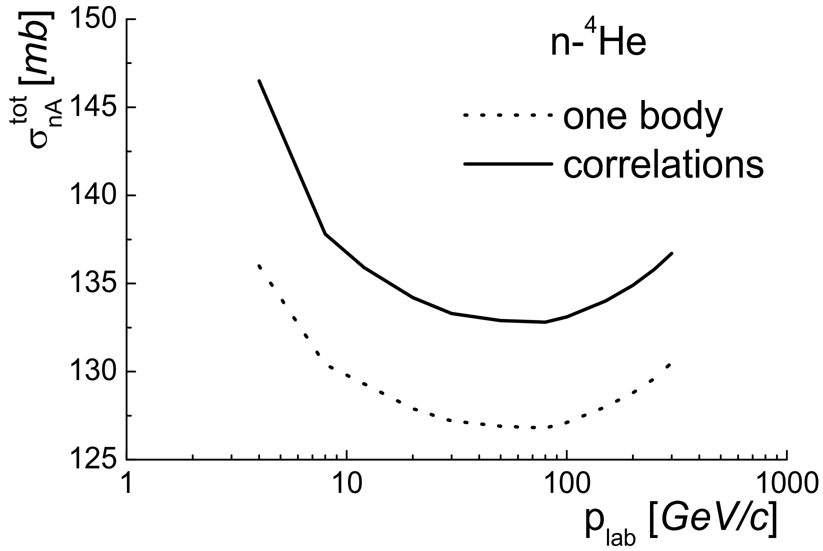
in the limit $A >> 1$:

$$\mathbf{e}^{\mathbf{i}\,\chi_{\mathbf{opt}}(\boldsymbol{b}_\mathbf{n})} \simeq \overbrace{exp\left[-A\int d\boldsymbol{r}_1\,\rho(\boldsymbol{r}_1)\,\Gamma(\boldsymbol{b}_n-\boldsymbol{s}_1)\right.}^{one-body\ term} +$$

$$\underbrace{\left. + A^2\,\frac{\int d\boldsymbol{r}_1 d\boldsymbol{r}_2\,\boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{1},\boldsymbol{r}_\mathbf{2})\,\Gamma(\boldsymbol{b}_n-\boldsymbol{s}_1)\,\Gamma(\boldsymbol{b}_n-\boldsymbol{s}_2)}{1-\int d\boldsymbol{r}_1\,\rho(\boldsymbol{r}_1)\,\Gamma(\boldsymbol{b}_n-\boldsymbol{s}_1)}\right]}_{two-body\ term}$$

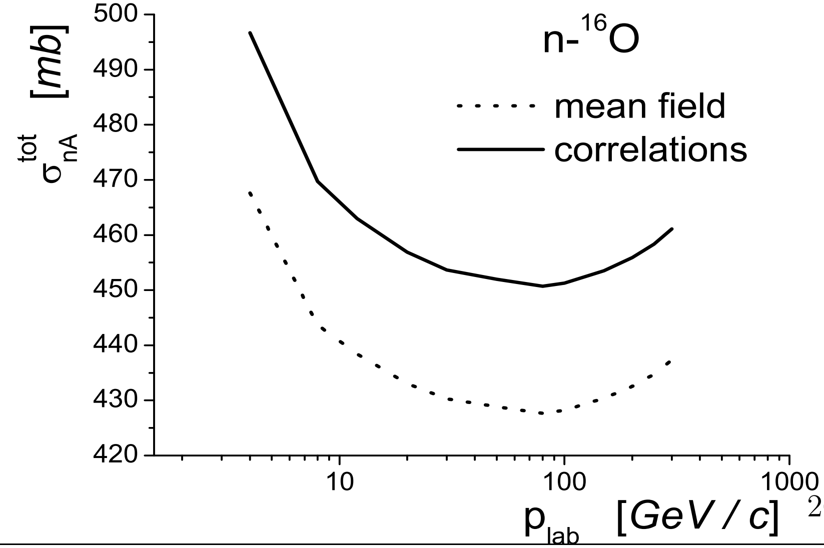
III.1 - *Results for* σ_{nA}^{tot}

"exact" wave function (^4He)



W.F from
Morita
et al.
Pr.Th.Phys.
18(1987)

cluster expansion (^{16}O)



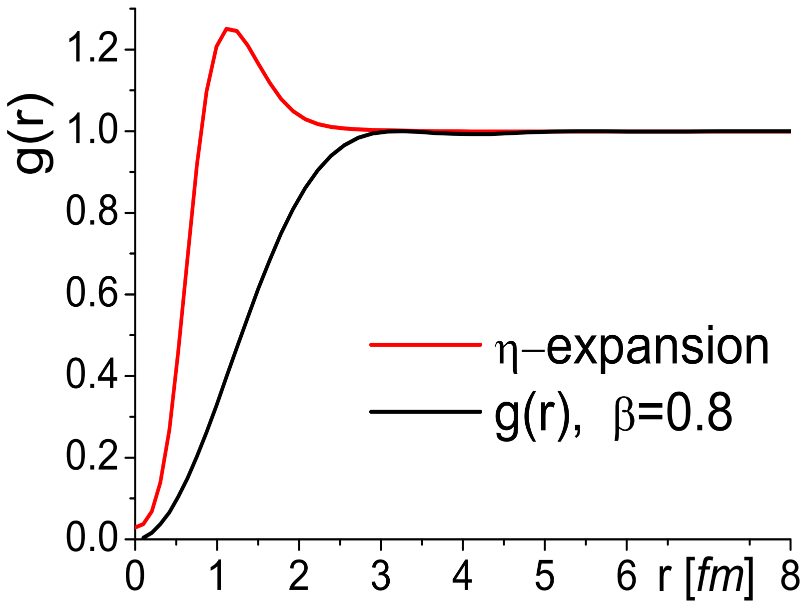
W.F. from
present work

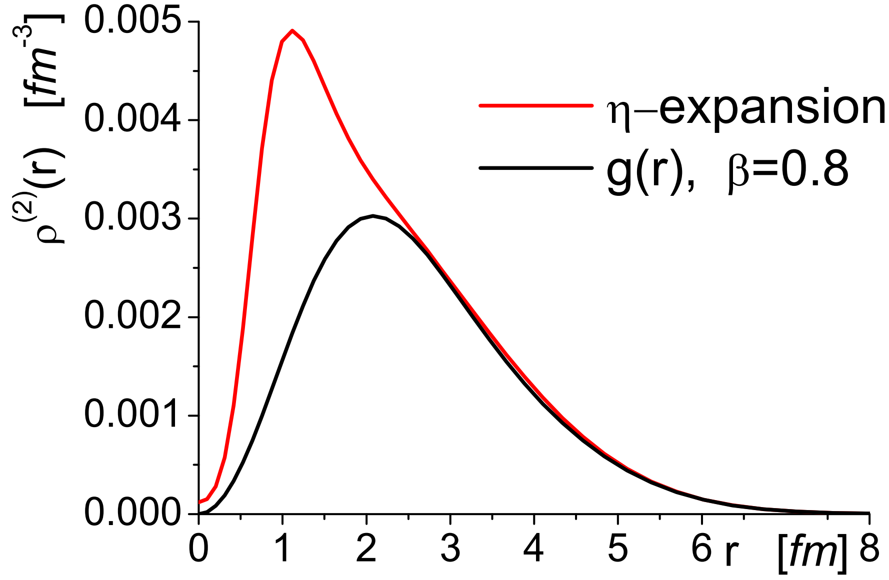
III.2 - *Correlations for nuclei other than ^{16}O and ^{40}Ca*

- in the case of ^{16}O we used $\rho^{(2)}_{A=16}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ from the $\eta-expansion$;
- for other nuclei we used the approximation:

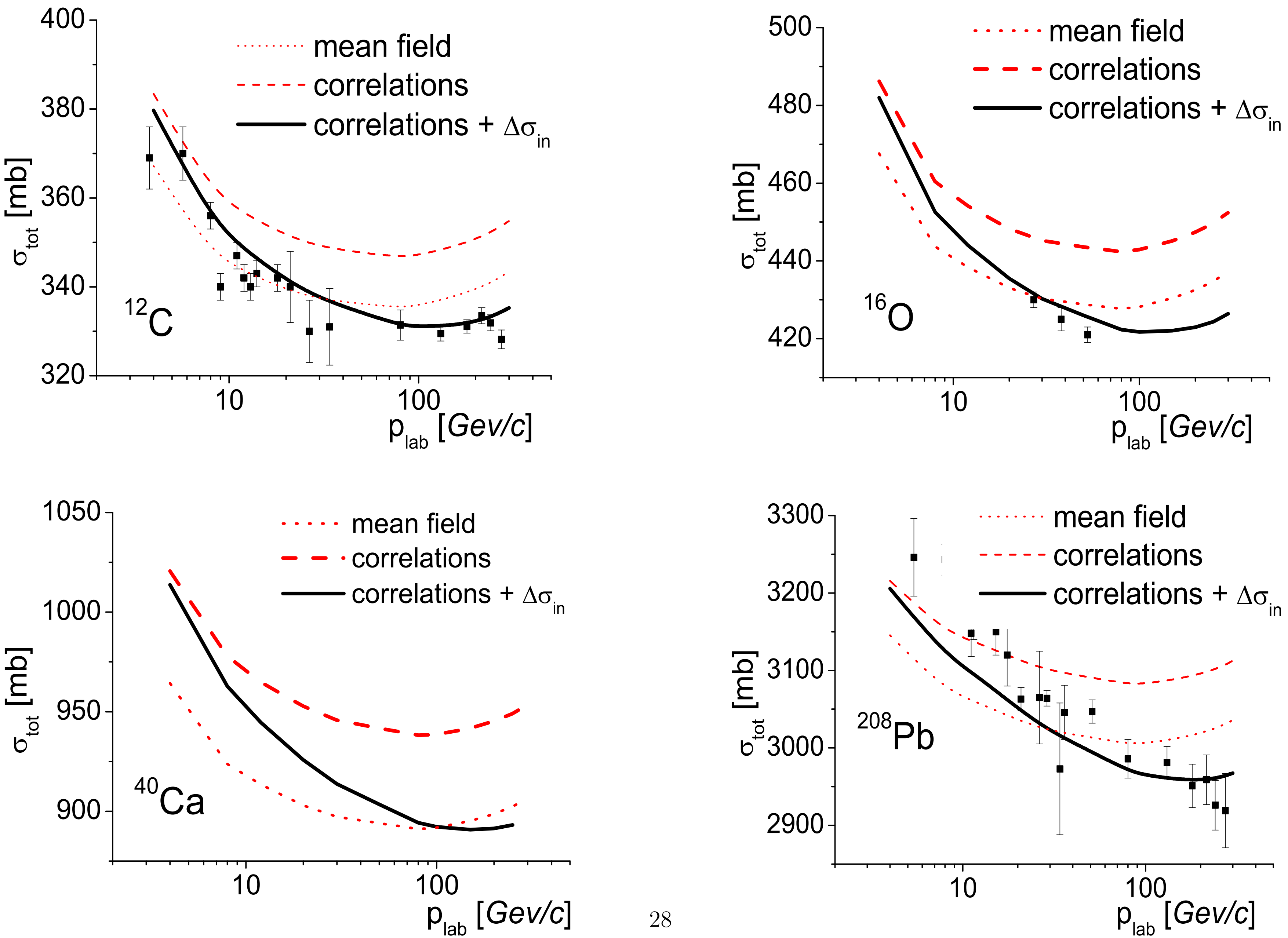
$$\rho^{(2)}_A(\boldsymbol{r}_1, \boldsymbol{r}_2) \simeq g(|\boldsymbol{r}_{12}|)\, \rho_A(\boldsymbol{r}_1)\, \rho_A(\boldsymbol{r}_2)\,; \qquad g(r) = 1 - exp(-\beta\, r^2)$$

with the function $g(|\boldsymbol{r}_{12}|)$ fixed from the case of ^{16}O:





III.4 - Results for $\sigma_{\mathbf{nA}}^{\mathbf{tot}} + \Delta\sigma^{\mathbf{in}}$



Conclusions

- we have developed a new *linked cluster expansion* for the calculation of the ground state properties of A interacting hadrons
- the method provides results comparable with the summation of a very large class of Meyer diagrams (FHNC)
- with the obtained ψ_o we have calculated semi-inclusive lepton scattering on *complex nuclei*, studying nuclear transparency, finite formation time effects, and the total $n - A$ cross section σ_{nA}^{tot}
- calculation of other inclusive reactions, *i.e.*, two-nucleon emission processes, are in progress.